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ON THE REGULARITY IN THE DISPOSITION OF THE PLANETS IN THE SOLAR SYSTEM

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A comparative analysis of the regularity in the disposition of the planets of the Solar system and of Jupiter's satellites is carried out. The existence of characteristic time and space scales, defined by the fundamental constants, is postulated.

The regular disposition of the planets in the Solar system was noticed long ago (see, e.g., [1, 2]).

As early as 1766 J. D. Titius proposed an empirical rule, according to which the mean distance (the semimajor axis) of the planet to the Sun is approximately described by the formula

$$a_n = 0.4 + 0.3 \times 2^{n-2}. \tag{1}$$

Here a_n is measured in astronomic units (a.u.) and n is an integer.

This rule is called the Titius-Bode law. It had played a definite role in the discovery of Uranus and of the belt of asteroids, which are at the distance of $a_5 = 2.8$ a.u. from the Sun, where, according to (1), a planet had to be found.

In spite of the wide popularity of the Titius-Bode law, its physical essence remains still unclear. There exists an extensive bibliography dealing with the interpretation of the above-stated regularity. A detailed list of literature sources is also given in [3–8]. Nevertheless, the question remains as open as before.

In this communication we shall consider the regularity existing in the disposition of the planets of the Solar system from a somewhat different standpoint.

The essence of this approach is as follows.

If we compare the most important physical characteristics of the planets, their mass and momentum (Table 1) [1], it is easy to see that practically all the mass and orbital momentum of the planets of the Solar system are concentrated in four giant planets. As to the overall characteristics of the planets of the Earth's group, they are negligibly small. In other words, from the point of view of an external observer, it is just for these giants, which are in essence typical representatives of the Solar system, that the regularity of the disposition must be observed, if it takes place.

To make sure that such regularity exists, we shall do as follows.

We shall combine all the planets of the Earth's group into one group and assign number $n = 1$ to it; index $n = 2$ will be assigned to Jupiter; index $n = 3$ will be assigned to Saturn; etc.

The relationship between the revolution period of the planet and its index n, presented in Fig. 1, is dramatic enough. This relationship is described by the expression

$$T_n^{1/3} = bn, \tag{2}$$

where n is an integer. Here T_n is the revolution period, measured in years, of the planet whose index is n; b is a non-dimensional constant.

Also valid is the relation

$$a_n^{1/2} = bn, \tag{3}$$

where a_n is the semimajor axis of the planet, b is the same constant as in (2). Formula (3) follows from (2) and from Kepler's third law.

We would like to draw attention to the fact that relations (2) and (3) point to the momentum of the planets being "quantized". Indeed, for the specific momentum $\widetilde{L_n}$ we have $\widetilde{L_n} = L_n/M_n$, where L_n is the

Table 1

Planet	Mass	Momentum	Revolution period
Mercury	0.055	0.03	0.241
Venus	0.815	0.69	0.616
Earth	1.00	1.00	1.00
Mars	0.107	0.13	1.881
Jupiter	317.82	725.0	11.86
Saturn	95.28	294.0	29.46
Uranus	14.56	64.0	84.01
Neptune	17.28	95.0	164.8
Pluto	0.0017	0.21	247.7

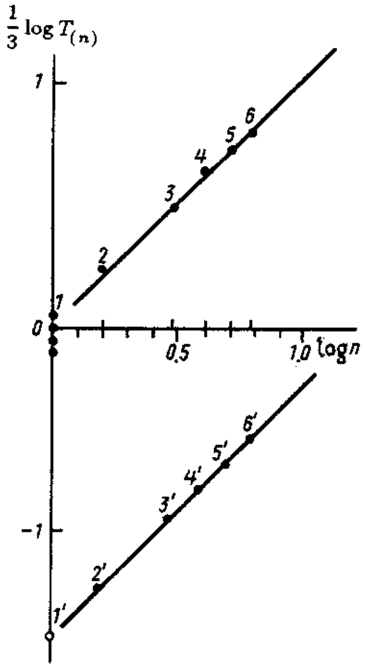

Fig. 1

Revolution period T_n of the planets and Jupiter's satellites versus number n: planets of the Earth's group (1), Jupiter (2), Saturn (3), Uranus (4), Neptune (5), Pluto (6), Amalthea (2′), Io (3′), Europa (4′), Ganymede (5′), Callisto (6′).

orbital momentum of the planet with the index n, whose mass is M_n. Taking into account the fact that for the planets of the Solar system the eccentricity of their orbits is small [1, 2], we obtain the estimate which follows from (2), (3):

$$\widetilde{L_n} = \frac{L_n}{M_n} \simeq \frac{M_n v_n a_n}{M_n} \sim \frac{a_n^2}{T_n} = bn. \tag{4}$$

Here v_n is the orbital speed of the nth planet.

The obtained relationships (2)–(4) hold true not only for the planets revolving around the Sun as the central attracting body. Even a superficial analysis shows that the above-stated regularity is observed also for the next in magnitude representative of the Solar system: Jupiter with its satellites.

As is known, Jupiter has five large satellites, whose masses are large (comparable, for instance, with the mass of the Moon or Mars). The discovery of four satellites dates back to Galileo. The satellite nearest to the surface of Jupiter, Amalthea, was discovered later. The remaining satellites are, apparently, captured asteroids, which became Jupiter's satellites comparatively recently, at considerably more recent stages of the existence of the Solar system [1, 2, 9].

The relation between the revolution period of Jupiter's satellites and their index n on a logarithmic scale can be represented by a straight line having the same inclination as for the planets of the Solar system, if index $n = 2$ is assigned to Amalthea (see Fig. 1).

The presence of a vacancy with $n = 1$ gives grounds to suppose that in the not distant past Jupiter had a satellite, which has fallen by the present moment. The presence of the Great Red Spot as a manifestation of Taylor's column [7] may be an evidence in favor of this supposition.

The mass of Jupiter is approximately three orders of magnitude smaller than the mass of the Sun, and therefore the revolution periods of Jupiter's satellites lie on the T'_n dependence curve, which runs in Fig. 1 below the T_n dependence curve for the Sun. This suggests the existence of the characteristic time scale T_* and characteristic spatial scale R_*, whose values are defined by the fundamental constants

$$T_* = 2\pi\frac{\gamma m}{c^3}\left(\frac{M}{m}\right)^{s_1}; \quad R_* = \frac{\gamma m}{c^2}\left(\frac{M}{m}\right)^{s_2}. \tag{5}$$

Here γ is the gravitational constant, M is the mass of an attracting body, c is the speed of light, m is the fundamental mass. The problem of the fundamental mass will be discussed below.

For formulas (5) to be consistent with Kepler's laws, it is necessary that the power exponents s_1 and s_2 should meet the condition

$$2s_1 - 3s_2 = -1 \tag{6}$$

or

$$s_1 = 1 + \varepsilon/2; \quad s_2 = 1 + \varepsilon/3. \tag{7}$$

It is easy to verify that from the relation

$$\frac{1}{3}\log T_n = \frac{1}{3}\log T_* + \log n, \tag{8}$$

which follows from (2) and (5), the value of the factor ε can be estimated:

$$\frac{1}{3}\log T_n^{\mathrm{Sun}} - \frac{1}{3}\log T_n^{\mathrm{Jup}} = \frac{s_1}{3}\log\{M^{\mathrm{Sun}}/M^{\mathrm{Jup}}\}. \tag{9}$$

From Fig. 1 it follows that the value of the left-hand side of the equality is about 1.3. Taking into account the estimate $M^{\mathrm{Sun}}/M^{\mathrm{Jup}} \sim 10^3$ [1], we obtain $s_1 \simeq 1.3$ ($\varepsilon \simeq 0.6$–0.7).

Now we can pose the question about the fundamental mass m. Using $M^{\mathrm{Sun}} \simeq 2 \times 10^{33}$ g, $r_g = \gamma M^{\mathrm{Sun}}/c^2 \simeq 1.49 \times 10^5$ cm, $t_g = 2\pi r_g/c \simeq \pi \times 10^{-5}$ s, 1 year $\sim \pi \times 10^{12}$ s, we obtain from expression (5) the estimate

$$\pi \times 10^{12} \simeq \pi \times 10^{-5}\left(\frac{2 \times 10^{33}}{m}\right)^{s_1 - 1}, \tag{10}$$

i.e.,

$$m \simeq 2 \times 10^{33-34/\varepsilon}\ \mathrm{g}. \tag{11}$$

If $\varepsilon \simeq 0.6$, then $m \simeq 2 \times 10^{-24}$ g by the order of magnitude is comparable with the mass of the proton $m_p = 1.67 \times 10^{-24}$ g.

The obtained result shows that the regularity of the order, pointed out in this work, probably [10], has a substantially more profound significance than might appear at first glance.

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