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ON THE THEORY OF HOPPING CONDUCTIVITY OF QUASI-ONE-DIMENSIONAL DISORDERED SYSTEMS

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The anisotropy of hopping conductivity of quasi-one-dimensional systems related to the convoluted shape of optimal carrier paths is considered. It is demonstrated that for systems described in the framework of the R-percolation model the specific features of the carrier path shape do not lead to exponential anisotropy of the conductivity.

It is well known that the conductivity of disordered systems can be often evaluated using methods of the percolation theory. In particular, the problem of finding the hopping conductivity for a system of sites with random positions and energies can be reduced to the bond problem on random sites. The main exponential temperature and concentration dependences of the system conductivity in this case are determined by the critical value of the transition rate between localized states, corresponding to the appearance in the system of an infinite cluster of conjugated bonds. Owing to the fact that with a uniform distribution of local centers (sites) in space the percolation threshold is unique even for a problem with anisotropic transition rates, in this case there is no anisotropy of the exponential factor of the conductivity [1]. At the same time, for systems with a nonuniform spatial distribution of sites the situation can be essentially different. In quasi-two-dimensional systems, two percolation thresholds may exist, giving rise to the exponential anisotropy of the hopping conductivity [2]. For quasi-one-dimensional systems with a weak coupling between the wires, for which the percolation threshold is unique, different exponential temperature variations for the longitudinal and transverse hopping conductivities were also obtained in [3]. Using the percolation approach, we shall discuss the possibilities of the existence of exponential anisotropy of the hopping conductivity for some model quasi-one-dimensional systems with a unique percolation threshold.

Specific features of the situation considered can be illustrated with a well-known model of anisotropic percolation on a regular lattice [4]. For a two-dimensional anisotropic system the percolation threshold is unique; the boundary of the percolation region on the plane $(\eta_\parallel, \eta_\perp)$ is determined by the equation

$$\eta_\parallel + \eta_\perp = 1, \tag{1}$$

where $\eta_\parallel, \eta_\perp$ are the probabilities of formation of longitudinal and transverse bonds. The probability of formation of a chain of s longitudinal bonds is $P_s = \eta_\parallel^s(1-\eta_\parallel)$ $(\sum_s P_s = 1)$ and the average number of bonds in the chain is

$$\bar{s} = \sum_s sP_s = \eta_\parallel(1-\eta_\parallel)^{-1}.$$

For one-dimensional systems there is no percolation (for $\eta_\perp = 0$ one has $\eta_\parallel = 1$); percolation appears when the number of transverse bonds is sufficiently large. The average number of transverse bonds joining a longitudinal chain with neighboring ones is $\nu_\perp = 2\bar{s}\eta_\perp$; this equation takes into account that the number of neighboring wires is two. The percolation sets in when $\nu_\perp = \nu_c$; according to Eq. (1), $\nu_c = 2$.

The onset of percolation both in longitudinal and in the transverse directions is due to the inter-chain transitions. For a strongly anisotropic system $(\eta_\perp/\eta_\parallel << 1)$ the average length of the longitudinal fraction of a chain is large and the average number of transverse bonds is small, so the percolation paths include prolonged longitudinal sections with rare transverse connections. Accordingly, near the percolation threshold the shortest chains of bonds connecting the opposite boundaries of a macroscopic sample have markedly different lengths in the longitudinal and transverse directions. Assuming that the bonds correspond to resistances of equal magnitude, one finds that the transverse resistance is about $(1-\eta_\parallel)^{-1}$ times greater than the longitudinal one.

One might expect that the specific shape of percolation paths considered above would also be of importance for problems of hopping conductivity. Consider a simple model of a quasi-one-dimensional system with sites randomly distributed on wires that are periodically arranged in a plane. The problem of hopping conductivity of such a system is equivalent to the calculation of the overall resistance of a Miller–Abrahams random network of resistances R_{mn} connecting the sites,

$$R_{mn} = R_0 \exp\{2r_{mn}/r_0\},$$

where R_0 is the pre-exponential factor taken to be constant for simplicity, r_0 is the localization radius, and r_{mn} is the inter-site distance. Let n be the density of site distribution along the wires and the inter-wire distance d is greater than the average inter-site distance on a wire n^{-1}. Then the problem is reduced to the bond problem, a bond being defined for a given r by $r_{mn} < r$. For a quasi-one-dimensional system the percolation depends on the presence of transverse bonds between the wires; for $r < d$ there are no such bonds and there is no percolation. The infinite cluster of conjugated bonds appears only for $r > d$. The percolation threshold can be then determined from the condition $\nu_\perp = \nu_{cr}$, where $\nu_\perp$ is the average number of transverse bonds connecting the fragment of the path lying on the wire with neighboring wires and ν_{cr} is the critical value of this number for a problem with random sites. Since the average length of such a fragment is large, its resistance can turn out to be large compared to the resistance of transverse bonds.

Using the Poisson distribution for sites on the wires, one can find:

the probability of finding a chain of s sites with nearest-neighbor distances smaller than r, $P_s = [1 - \exp\{-nr\}]^s \exp\{-nr\}$;

the average number of sites in the path fragments on the wires,

$$\bar{s} = \sum_s sP_s = [1 - \exp\{-nr\}]\exp\{nr\}$$

(it is exponentially large for $nr >> 1$);

the average resistance connecting the neighboring sites on a wire,

$$\bar{R} = n\int dr\, R_0 \exp\{2r/r_0\}\exp\{-nr\} = R_0 \exp\{2r/r_0\}\frac{nr_0}{nr_0 - 2}\,\frac{\exp\{-nr\} - \exp\{-2r/r_0\}}{1 - \exp\{-nr\}}.$$

The average number of transverse bonds for the considered fragment of the chain of bonds is $\nu_\perp = 4\bar{s}n\sqrt{r^2 - d^2}$. The condition $\nu_\perp = \nu_{cr}$ determines the percolation threshold when the infinite cluster appears:

$$r_c^2 = d^2 + (\nu_{cr}^2/16n^2)[1 - \exp\{nr_c\}]^{-2}\exp\{-2nr_c\}.$$

Thus

$$\varepsilon = r_c - d \cong (\nu_{cr}^2/32dn^2)\exp\{-2nd\}$$

(this expression is valid for $2n\varepsilon \ll 1$). One can see that for $nd \gg 1$ the critical value r_c is close to d and the resistance of transverse bonds is close to $R^{(tr)} = R_0 \exp\{2d/r_0\}$.

The shape of the conduction paths, lying in the infinite cluster, is similar to that considered above for the problem of anisotropic percolation on a lattice. One might expect large anisotropy when the total resistance of a fragment of the chain of bonds lying on the wire

$$R^{(l)} = \bar{s}\bar{R} = R_0 \exp\{2d/r_0\}[nr_0/(nr_0 - 2)][\exp\{(n - 2/r_0)d\} - 1]$$

exceeds the resistance of a transverse bond $R^{(tr)}$. This is true when $nr_0 \gg 1$. In this case in the immediate neighborhood of the percolation threshold the resistance of the transverse percolating chain of bonds lying inside the infinite cluster is greater than the resistance of a longitudinal chain with the same distance between its ends by $[nr_0/(nr_0 - 2)][\exp\{(n - 2/r_0)d\} - 1]$ times. One can show however that for the model considered this does not give rise to exponential anisotropy of the conductivity. In fact, the total resistance of a transverse chain of bonds is appreciably decreased when it is straightened by transverse resistances connecting sites of neighboring wires with inter-site distances of the order $\tilde{r} = \sqrt{d^2 + An^{-2}}$, where A is a constant of the order of unity. As $nd \gg 1$, one has $\tilde{r} \cong d + A/(2n^2d)$ and the resistances of the shortcutting

bonds differ from $R^{(tr)}$ by a factor of $\exp\{A/n^2 r_0 d\}$. As $nr_0 \gg 1$, those resistances should also be included into the critical subnetwork [1] which is defined by the conditions $d < r < d + r_0$, and there is no exponential anisotropy of the conductivity. This indicates that the procedure of independent optimization of transverse hops used in [3] is inapplicable to the systems under consideration described in the framework of the model of R-percolation.

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