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PHYSICAL FOUNDATIONS OF MILLIMETER AND SUBMILLIMETER DEVICES.
Part 1. OPEN RESONATORS AND OPEN WAVEGUIDES*

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Theoretical and experimental data of mathematical and physical modeling are described for open resonators and open waveguides used as principal components for devices operating in millimeter and submillimeter wavelength bands. Using the methods of operator-function theory and some techniques from the theory of Morse's critical points, the fundamental properties of the spectrum of natural vibrations and waves and electrodynamic characteristics of open systems studied are established, and physical phenomena and effects are adequately described. Theoretical and experimental data are compared.

Theoretical, experimental, and applied research in the field of millimeter (mm) and submillimeter (sub-mm) wavelengths which has been carried out systematically at the Institute of Radiophysics and Electronics of the Ukrainian Academy of Sciences for nearly thirty years have shown that this wavelength range can be almost exhaustively mastered with the aid of open electrodynamic structures (OS).

Efficient OS include open resonators (OR), open waveguides (OW), and diffraction gratings (DG); they make it possible to concentrate and diffuse electromagnetic fields at specified space domains, to propagate and radiate them directedly, to transform surface waves into bulk ones and vice versa.

In order to master the mm and sub-mm waves, new OS types [1–4] are required that would be radically different from those used at the first stage of radio engineering development and would be applied on a qualitatively new physical foundation. Such structures are naturally connected with the entire space, comparable with the wavelength in size, and have various configurations of the boundary surfaces. The study of the propagation, scattering, absorption, and transformation of inherent fields by such OS involves the construction of rigorous mathematical methods for solving electrodynamical boundary-value problems, the development of new experimental techniques, and an application of the theoretical and experimental data obtained which is quite different from the classical approach based on the scaled modeling method.

1. OPEN RESONATORS.
THEORY

The theoretical studies of OR were long performed by asymptotic and heuristic methods. The models of open resonators built in the long- and short-wave approximations described just a portion of their spectra. These approaches become inapplicable for the analysis of the OR spectral characteristics and forced oscillations when the OR size is commensurable with the wavelength.

Open resonators fundamentally differ from closed resonators by radiation losses, by their multiply-connected cross sections, and by the presence of ribs. One also has to describe adequately the behavior of the inherent electromagnetic field scattered by them at infinity. Therefore the spectrum of natural OR vibrations is no longer real. Additional requirements arise to the energy relations in various space domains

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for the formulation of boundary-value problems, and the very nature of these problems changes: they form a new class of non-self-consistent spectral problems in whose conditions the spectral parameter usually enters nonlinearly. These nonlinear spectral problems of mathematical physics call for new solution techniques and, in particular, for the application of the operator-function method for a single or multiple complex variables. This method binds together the homogeneous (spectral) and inhomogeneous (excitation-modeling) electrodynamic problems, because one can simultaneously determine the localization domains of the spectrum and the resolvent set of operators, thus proving the existence and uniqueness of the solution.

Let us discuss in more detail the construction of the OR spectral theory. It is well known that in classical OR diffraction problems the theorem of uniqueness holds and the frequency is a real quantity. No simple method for the description of OR resonance effects was earlier available. This difficulty was overcome by a mathematically quite correct "analytical extension" of the diffraction boundary-value problem to the domain of complex frequencies [2, 3, 5]. In order to do this in the two-dimensional case for bounded OR mirrors, one must replace the Sommerfeld condition at infinity by the Reichardt conditions [5, 6]. The boundary-value problem for real frequencies is "analytically extended" to the appropriate Riemann surface for complex frequency values. The problem has a unique solution everywhere on the Riemann surface, except for some probably empty and at most countable discrete set of frequencies with a single possible point of accumulation at infinity. The eigenvalues of the homogeneous problem in question, to which its eigenfunctions correspond, are simultaneously the finite-multiplicity poles for the analytical extension of the operator resolvent in the original diffraction boundary-value problem [5]. The mathematical apparatus for this spectral OR theory is the theory of meromorphic operator-functions. Thus, the eigenvalue problem with the Reichardt condition is reduced to a spectral problem for an operator-function, that is, to finding the set of complex frequencies (and eigenfunctions) at which the homogeneous equation has a nontrivial solution. What is important, this spectral problem allows an efficient numerical solution.

This approach, based on using the regularization procedure for solving spectral problems, was implemented for the first time in [7] for coordinate OR problems with various inclusions. This method was also extended to noncoordinate problems which equivalently correspond to certain integral and integro-differential equations [8]. Now the dispersion relations produced in the complex domain of frequency variation acquire for OR a clear mathematical meaning: their complex roots correspond to eigenfunctions which determine the resonance behavior of scattered fields for frequencies with small imaginary parts (the frequencies which lie at the physical sheet; a detailed analysis shows that frequencies located at other sheets of the Riemann surface also play an important role).

We must stress a fundamental distinction between the solution methods for diffraction and spectral boundary-value problems. A correctly posed diffraction boundary-value problem has a unique solution. Quite a different situation arises in the solution of a spectral problem. No theorem of uniqueness exists for a spectral problem. Moreover, a solution to a homogeneous spectral problem is sought exactly where the theorem of uniqueness is violated. The question arises whether the original spectral boundary-value problem is equivalent to the spectral problem for the homogeneous equation which follows from the inhomogeneous equation of diffraction theory. We were able to prove this for OR and to study spectral problems with the aid of the operator-function which arises in the diffraction problem.

The spectral-theory (and diffraction-theory) methods which we describe are now a powerful tool for the analysis of the physical properties of various intricate OR.

As an example, let us consider a simplest spectral problem for a cylindric OR formed by two-dimensional circular metal screens with a dielectric inclusion whose axis is parallel to the mirrors' generatrices. We have to find such values of the spectral parameter $\varkappa \in \Lambda$ (Λ is the Riemann surface of the analytical extension of the fundamental solution to the two-dimensional Helmholtz equation with respect to the variable $\varkappa = ka$, where $k = \omega/c$, ω is the fundamental frequency, $S = \bigcup_{j=1}^{3} S_j$ is the OR cross section by the coordinate plane, where S_1 and S_2 are the mirrors' cross sections and S_3 is the inclusion cross section; c is the velocity of light in vacuum; $a = \max(a_1, a_2)$; a_1 and a_2 are the mirrors' radii), at which a nontrivial solution to the homogeneous Helmholtz equation exists and satisfies the homogeneous boundary conditions, the conjugation conditions, a Meyxner-type condition, and the Reichardt condition at infinity.

The Riemann surface is $\Lambda = \bigcup_{j=-\infty}^{\infty} \Lambda_j$, where $\Lambda_j = \{-\pi + 2\pi j < \arg \varkappa < \pi + 2\pi j\}$. Let us call Λ_0 the physical sheet. Since the time dependence is selected in the form $\exp\{-i\varkappa c/a\}$, then for $\varkappa \in \Lambda_0$ $\operatorname{Im}\varkappa < 0$, which corresponds to vibrations that decay exponentially with time. All other sheets Λ contain frequencies with $\operatorname{Im}\varkappa < 0$ and $\operatorname{Im}\varkappa > 0$, to which correspond vibrations decaying and growing with time, respectively.

The function $u(x, y)$ describes the longitudinal component of the inherent electromagnetic field H_z for the H-polarization and E_z for the E-polarization. Defining the general solution to the boundary-value problem for H-vibrations and limiting ourselves to the case where the OR mirrors are concave in the same direction, utilizing the boundary conditions at the mirrors and at the interface of the OR internal and external spaces, and applying to the resulting functional equations the regularization procedure by the method employed in the Riemann–Hilbert problem [9], we come to an infinite homogeneous system of linear algebraic equations in terms of the Fourier components of the desired field of natural vibrations. The matrix elements of this system are functions of the spectral parameter $\varkappa$. By replacing the infinite system with a finite one we can find approximate fundamental frequencies, which are the roots of the truncated system's determinant. It can be proved that the matrices of the algebraic equations specify nuclear operators in the space l_2 for any $\varkappa$. These operators are denoted by $A^{ns}(\varkappa)$, and the truncated equations are

$$x^n = \sum_{s=1}^{3} A^{ns}(\varkappa)x^s \quad (n = 1, 2, 3). \tag{1}$$

The system of operator equations (1) represents an eigenvalue problem for some matrix operator-function. In correspondence with the system of operator-functions $\{A^{ns}(\varkappa)\}$ can be put an operator-function $A(\varkappa) = \|A^{ns}\|^3_{n,s=1}$, acting in the space l_2^3 [2]. Finally, (1) is written as an operator equation in l_2^3:

$$\{I - A(\varkappa)\}x = \Theta, \tag{2}$$

where I is the unity operator in l_2^3 and Θ is the zero element. It can be proved [5] that there exists a nontrivial solution to Eq. (2), which in turn shows the equivalence of the existence of a nontrivial solution to both the infinite system of linear algebraic equations and to the original spectral problem in the space l_2. Thus, the spectrum of OR natural electromagnetic vibrations coincides with the eigenvalue set of the operator-function $I - A(\varkappa)$. The following statement is true.

Natural two-dimensional electromagnetic vibrations of OR form a discrete, finite-multiplicity spectrum, i. e., a countable isolated set with a single condensation point at infinity, and it is located in the region $\operatorname{Im}\varkappa < 0$.

The open resonators in question may possess high-Q vibrations whose fundamental frequencies may be located arbitrarily close to the real axis. Moreover, one of the most important implications of the strict spectral OR theory is the opportunity of finding the Morse critical points of the dispersion equations

$$\mathcal{F}(\varkappa) \equiv \det\{I - A(\varkappa)\} = 0 \tag{3}$$

and the corresponding intertype vibrations which can be used to determine the eigenvalues of the operator $A(\varkappa)$ [10, 11]. In this case $A(\varkappa)$ also depends on nonspectral parameters, such as $\chi = b/a$, where b is the OR aperture. Now $A(\varkappa)$ is written as $A(\varkappa, \chi)$, where χ varies in the analyticity domain of $A(\varkappa, \chi)$, $\mathcal{D}_\chi \in \mathbf{C}$ ($\mathbf{C}$ is the complex plane). Let $\varkappa \in \Lambda_0$, where $\Lambda_0 = \{-\pi < \arg\varkappa < \pi; \varkappa \neq 0\}$ is the zero sheet of the Riemann surface Λ. Since $A(\varkappa, \chi)$ is a nuclear and analytical operator-function, the set of zeros of Eq. (3) coincides with the set of eigenvalues $\sigma(\varkappa)$ of the operator-function $I - A(\varkappa, \chi)$ ($\sigma(\varkappa)$ is defined on the set $\Lambda_0 \times \mathcal{D}_\chi$). It is important to find out what occurs with $\sigma(\varkappa)$ as $\varkappa$ is varied in $\mathcal{D}_\chi$.

For this purpose, let us introduce an analytical set $\mathcal{D}$ of the form $\sigma_0 = \{(\varkappa, \chi) \in \mathcal{D} : \mathcal{F}(\varkappa, \chi) = 0\}$, and regard $\mathcal{F}(\varkappa, \chi)$ as the mappings $\mathcal{F} : \mathbf{C}^2 \to \mathbf{C}$ defined on $\mathcal{D}$ (χ is a complex quantity). If an isolated singular point $(\varkappa_0, \chi_0)$ of the mapping $\mathcal{F}$ exists, the local structure of the set σ_0 in the vicinity of $(\varkappa_0, \chi_0)$ is determined by the location and type of $(\varkappa_0, \chi_0)$, while away from $(\varkappa_0, \chi_0)$ the set has the local structure of a hyperplane. Clearly, small variations in χ around $(\varkappa_0, \chi_0)$ give rise to small changes in $\sigma(\varkappa)$.

Intertype vibrations have a peculiar dispersion law, with a special behavior of the two fundamental frequencies as functions of χ at $\operatorname{Im}\chi = 0$ (the Wien graph). The naturally arising task is to describe, in terms of regular and singular points of the mapping $\mathcal{F}$, the conditions which correspond to this dispersion law. The hyperplane local structure of the set σ_0 does not correspond to this law. Therefore let us consider the set of critical points $\sigma_{\text{cr}} = \{(\varkappa, \chi) \in \mathcal{D} : \operatorname{grad} \mathcal{F} := (\partial\mathcal{F}/\partial\varkappa, \partial\mathcal{F}/\partial\chi) = 0\}$ and the set of isolated Morse critical points

$$\sigma_{M,\text{cr}} = \left\{ (\varkappa, \chi) = \sigma_{\text{cr}} : \frac{\partial^2\mathcal{F}}{\partial\varkappa^2}\frac{\partial^2\mathcal{F}}{\partial\chi^2} - \left(\frac{\partial^2\mathcal{F}}{\partial\varkappa\partial\chi}\right)^2 \neq 0 \right\}.$$

Let $(\varkappa_0, \chi_0)$ be an isolated Morse critical point lying in the vicinity of σ_0 (i.e., $\mathcal{F}(\varkappa_0, \chi_0) = \delta$ is small). Then Eq. (3) can be expanded in a series around $(\varkappa_0, \chi_0)$ up to cubically small terms. Since $(\varkappa_0, \chi_0) \in \sigma_{M,\text{cr}}$, the substitution of the variables $\widetilde{\varkappa} = \psi_1(\varkappa, \chi)$, $\widetilde{\chi} = \psi_2(\varkappa, \chi)$ allows this series to be represented as a canonical quadratic form [10, 11]

$$\widetilde{\varkappa}^2 + \widetilde{\chi}^2 + \delta = 0, \tag{4}$$

whence $\widetilde{\varkappa} = \pm\sqrt{-\chi^2 - \delta}$. For $\delta = 0$ we have $\widetilde{\gamma} = \pm i\widetilde{\chi}$. Equation (4) defines the dispersion law (dependence of the spectral parameter, the frequency, on a nonspectral one, i.e., a or b) characterizing the intertype coupling of vibrations, while the quantity $\delta = \mathcal{F}(\varkappa_0, \chi_0)$ determines the extent of that coupling. For $\delta = 0$ at $(\varkappa_0, \chi_0)$, we have $\mathcal{F}(\varkappa, \chi)$ twice degenerate in both $\varkappa$ and χ.

The dependencies $\operatorname{Re}\varkappa(\chi)$ and $\operatorname{Im}\varkappa(\chi)$ in the physical variation domain of the nonspectral OR tuning parameter (i.e., for $\operatorname{Im}\chi = 0$) can be investigated with the aid of (4) for $\operatorname{Re}\widetilde{\varkappa}(\widetilde{\chi})$ and $\operatorname{Im}\widetilde{\varkappa}(\widetilde{\chi})$, with $\widetilde{\chi}$ varying along the line $\operatorname{Re}\widetilde{\chi} = \xi$, $\operatorname{Im}\widetilde{\chi} = \alpha\xi + \beta$, where ξ is the real parameter of the line and α is its inclination angle. The two-parameter (in α and β) family of curves $\operatorname{Re}\widetilde{\varkappa}(\xi)$, $\operatorname{Im}\widetilde{\varkappa}(\xi)$ includes, at certain α and β, dependencies similar to the well-known Wien graph. It is important to construct such dispersion curves which correspond to all possible qualitative dispersion laws near Morse's critical points at various α and β.

Thus, the existence of isolated Morse's critical points $(\varkappa_0, \chi_0)$ of Eq. (3) results in the existence around $(\varkappa_0, \chi_0)$ of two solutions, $\varkappa_+(\chi)$ and $\varkappa_-(\chi)$, to that dispersion equation. The behavior of these solutions with a varying OR tuning parameter χ is completely determined by Eq. (4) and describes intertype OR vibrations; in particular, it determines abrupt variations of the OR Q-factor (diffraction losses) after a small change in its geometric parameters [10, 11].

OR excitation. If a point monochromatic source is located at some point in space, then, if an OR with an inclusion is present, we can construct the Green function $G(\varkappa; r)$ (for the latter the conditions of the classical diffraction problem are known) and obtain an inhomogeneous infinite set of linear algebraic equations for the Fourier field components, which differs from (1), (2) only in its right-hand part and for the case of H-vibrations has the form

$$\{I - A(\varkappa)\}x = f. \tag{5}$$

Formally solving (5), we have $x = R_A(\varkappa)f$, where $R_A(\varkappa) = \{I - A(\varkappa)\}^{-1}$ is the resolvent of the operator-function $A(\varkappa)$. The solution $x = R_A(\varkappa)f$ is meaningful if for $\operatorname{Im}\varkappa > 0$ a bounded operator R_A exists. It can be proved that a solution to (5) exists and is unique if $A(\varkappa)$ for $\operatorname{Im}\varkappa > 0$ specifies a completely continuous operator in L and the homogeneous equation corresponding to (5) has only a zero solution.

The following statement is true.

The Green function $G(\varkappa; r)$ is analytic in $\varkappa$ for $\operatorname{Im}\varkappa > 0$ and admits analytic continuation to Λ, except only for the set of poles which coincides with the spectrum of problem (1), (2), i.e., with the set $\sigma(A)$.

Numerical modeling of OR vibrations. The rigorous mathematical theory of OR and their excitation by various sources, developed by us, makes it possible to perform efficient computer modeling (with arbitrary OR parameters and inclusion properties); this enables one to determine the boundaries of applicability of the heuristic methods employed for analyzing such structures and to predict new physical phenomena, which was virtually impossible with the methods known earlier.

Computer experiments allowed us to discover long-wave resonances for H-polarized vibrations of coaxial OR. For instance, for an OR with the geometric parameters $\theta_1 = \theta_2 = 5°$, $a_2/a_1 = 0.7$ we have $\varkappa_1 = 0.3563 - i0.0196$ and $\varkappa_2 = 0.6310 - i0.00001$ (here θ_1 and θ_2 are the slit widths, a_1 and a_2 are the radii of the coaxial OR). An analysis of the fields which arise when the OR is excited by a plane wave at the frequencies $\operatorname{Re}\varkappa_1$ and $\operatorname{Re}\varkappa_2$ shows that the longer-wave resonance at the frequency $\operatorname{Re}\varkappa_1$ corresponds to the slit resonance case, in which the magnetic field is uniform in the outer-mirror slit and the electric field is concentrated in its vicinity. The vibration at $\operatorname{Re}\varkappa_2$ corresponds to a very high Q (about 3×10^4), the resonance wavelength $\lambda_{\varkappa_2} = 2\pi a_1/0.631$ exceeding the OR diameter by a factor of five. An analysis of the fields of this resonance vibration reveals the existence of fundamentally new, strictly counter-phase, high-Q vibrations.

It was also found that for $|\varkappa| \sim 1$ a simple spectrum arises in the OR, whose fundamental frequencies can be numbered. A full analysis of the field configurations can also be performed [5, 7].

We studied in detail the spectral characteristics of double-mirror OR. In the mm and sub-mm bands, OR with $1 < |\varkappa| < 20$, $b/a \approx 1$ (b is the aperture radius, a is the mirror curvature radius) are usually employed. In this frequency range, a confocal OR ($l/a \approx 1$, l is the distance between the mirrors) has sharply pronounced resonance properties.

Let us dwell in more detail on intertype vibrations in a two-dimensional OR. The heuristic "bouncing ball" theory cannot predict this effect in principle. Let us consider the spectrum of natural H-polarized vibrations of a symmetric confocal OR ($a = l$). The spectrum of this OR was found to have four classes of symmetry. The intertype vibrations phenomenon is only possible for vibrations of the same symmetry class. For instance, H_{03} and H_{13} vibrations enter intertype coupling with H_{41} and H_{51} vibrations, respectively, when the parameter b/a is varied. As b/a grows, the Q-factor of H_{03} vibrations, which have three field variations along the OR axis, rises monotonically to $Q \sim 10^4$ ($b/a = 0.65$), then falls abruptly at $b/a \approx 0.775$, becoming the same as the Q-factor for H_{41} vibrations (the dispersion curve for $0.73 \leqslant b/a \leqslant 0.83$ is similar to the Wien graph and can be described by Eq. (4)). The Morse critical point for $b/a = 0.775$ corresponds to the H_{03} and H_{41} intertype vibrations of the OR in question [11].

EXPERIMENT

The properties of OR operating in the mm and sub-mm bands can be fully determined if theoretical conclusions are substantially supplemented by experimental data. In this connection, it should be first of all pointed out that the investigation of the OR characteristics calls for the development of new experimental techniques, quite different from those employed in the cm band for closed structures.

The holography-based [2, 3, 12] technique for electromagnetic field visualization [1] was employed for studying the phase and polarization characteristics of OR. This technique was implemented in a radioholographic setup [9, 12] based on a two-channel interferometer which allows the wave front of high-Q OR vibrations to be restored without perturbing the spatial structure of the excited vibrations. Synchronous recording of the amplitude and phase field characteristics was used as a basis for developing the resonance polarimetry method [1, 2, 12] in double-mirror mm and sub-mm OR and for designing an automatic device, a three-channel quasioptical interferometer with a second reference channel cross-polarized with respect to the first one.

Our techniques for measuring the amplitude, phase, and polarization characteristics of resonance OR fields enable one to obtain complete information on intricate OR. We shall list some of the results of such measurements which have been performed systematically for studying mm and sub-mm OR in diffraction electronics [1, 4], radiospectroscopy, and polarimetry [12]. The properties of hemispherical smooth-mirror OR were determined. The vibration spectra of such OR, their actual inherent Q-factors, and the visible field-distribution patterns in various OR cross sections were determined. In the 4-mm band, the maximum Q of semi-concentric OR reaches about 2×10^4. The case in which one of the flat mirrors in a hemispherical OR is covered by a grid, either fully or partially, was studied. In the first case, Q is reduced to about 5×10^3, in the second case to about 2.3–1.4 $\times 10^4$. The OR field distribution is especially sharply changed if the bottom mirror is partially covered by a grid. The formation processes of such fields were studied in detail, which is necessary for designing efficient OR in diffraction electronics, i.e., for creating diffraction radiation generators (DRG) in the mm and sub-mm wave bands [1, 4]. The hyperfine structure of such fields is important for this task — e.g., in the presence of a bounded grid at one of the OR mirrors or a coupling aperture in the upper mirror, etc. Such nonuniformities in the OR can be regarded as local phase filters which strongly perturb the ordinary OR vibrations, because the field amplitude is lower over the grid than over a smooth surface, and the E- and H-polarized fields differ greatly from each other. One can fully compensate for the phase variations introduced by the grid and achieve an increase in the amplitude of the H-polarized field near the grid, which is very important for improving the electron beam interaction with the grid surface waves. Thus, there arises an opportunity for controlling the OR amplitude-phase characteristics, which is especially necessary in designing specialized OR cells for mm and sub-mm spectroscopy [1–4].

Polarization measurements in OR became quite important in the studies on the dynamic polarization of atomic nuclei by mm waves [12].

Of special interest for mm-wave spectroscopy are OR with anisotropic films, employed for studying polarized nuclear target materials, as well as OR with total internal reflection prisms, dielectric layers with grids, etc. All these structures were studied in detail in [2, 3, 12], which allows one to speak of an emerging new science area, quasioptical mm-wave radiophysics for the investigation of dynamically polarized atomic nuclei.

2. OPEN WAVEGUIDES

The open waveguide (OW) is a promising type of integral channel for mm and sub-mm waves. It is a cylindric slot line, which in certain modes can also serve as an individual antenna system with a peculiar radiated field, or a miniature feed for a mirror antenna. The electrodynamic fields of this line are quasistatic, therefore it can be analyzed with the aid of equivalent circuits with appropriate values of capacitances, inductances, resistances, active and passive elements; the quasistatic character of the fields creates unique opportunities for component design on the basis of this channel. A special type of such lines, the mirror slot line, has a metal substrate over which an OW is arranged. Because the electric component of the OW field is concentrated in the slot region and the magnetic component exists inside the OW, the components whose operation depends on the electric field (mixers, detectors, *p-i-n* diodes, Gunn generators, etc.) must be arranged in the slot region, while the nonreciprocal components whose principle of operation relies on the magnetic field must be located inside the cylindric slot line, close to its wall. For a mirror slot line, many components can be arranged behind the metal shield. The small dimensions of the mm-band OW allow these components (and the OW itself) to be fabricated by the well-developed film technology [13].

The cylindric and mirror slot lines are very wide-band channels, with bandwidths of up to two octaves. In actual lines the attenuation amounts to fractions of a decibel per meter. As its slot width is increased, a cylindric slot line may smoothly pass from waveguide mode to radiating antenna system mode. Mirror slot lines have broader bandwidths and lower losses than cylindric ones.

THEORY

A mathematically rigorous wave propagation theory can be developed for cylindric or mirror slot-line OW in a way similar to what was described in Section 1. In the lines in question the slot width is much smaller than the wavelength. This means that the line is supercritical for all higher waves except the principal one. If a cylindric waveguide has a longitudinal slot, a zero (principal) wave exists in it. We shall call it the H_{00} wave. If the waveguide is filled with a dielectric, the slot wave becomes a hybrid one with prevailing H_z component, that is, a quasi-H_{00}-type wave. We shall not investigate here the properties of these slot lines by solving the boundary-value problem. Instead, we concentrate our attention on the analysis of their properties by the cross resonance method which is convenient for engineering calculation [2, 3].

Let us consider the oscillatory circuit for the quasi-H_{00} wave in a cylindric slot line: the slot electric field forms a quasy-lumped capacitance C_0, while the magnetic field inside the line plays the role of an inductive coil L_0.

Now C_0 and L_0, connected in series, form an oscillatory circuit, which can be used for studying the propagation of the H_{00} wave along the line by the cross resonance method. We take C_0 in the form [13] $C_0 = C_t + 2C_l$, where $C_t = d_2/d_1$ ($d_1 = a_s \sin\theta$, $2d_2$ is the metal substrate thickness, a_s is the radius, θ is the slot width); $C_l = (1/\pi)\ln(1/\sin 0.5\theta)$ is the edge capacitance. The inductance L_0 is calculated similarly to the case of an infinite waveguide. If the line contains N layers of dielectric, we introduce [13] the quantity $\varepsilon_{\rm ef} = \sum_{i=1}^{N} \varepsilon_i \nu_i$, where ε_i is the permittivity of the ith layer and ν_i is the line filling ratio for the ith dielectric [13]. For $N = 2$ we have

$$\varepsilon_{\rm ef} = \varepsilon\nu_1 + \nu_2, \quad \nu_2 = 1 - \nu_1 = [2 + Q_s(2\eta - Q_s)]^{-1}, \quad h = k\sqrt{\varepsilon_{\rm ef} - \frac{\pi}{2k^2\eta S_0}}, \tag{6}$$

where

$$Q_s = \frac{\pi d_2}{2a_s \sin\theta}, \quad \eta = Q_s + \ln\frac{1}{\sin 0.5\theta};$$

$S_0 = \pi a_s^2$ is the line cross-section area. Two characteristic wavelengths correspond to the dispersion equation (6): the critical wavelength $\lambda_{\rm cr}$ such that $h(k_{\rm cr}) = 0$, and the transition wavelength λ_t for which $h(\lambda_t) = k$,

$$\lambda_{\rm cr} = 2\pi a_s\sqrt{2\eta\varepsilon_{\rm ef}}, \quad \lambda_t = 2\pi a_s\sqrt{2(\varepsilon_{\rm ef} - 1)}. \tag{7}$$

For $\theta \approx 3°–10°$ we have $\lambda_c > 20a_s$. For $\lambda_0 < \lambda_t$, a slow wave propagates in the cylindric slot line with a phase velocity $v_\Phi < c/\varepsilon_2$, where ε_2 is the permittivity of the ambient space. The unimodal regime band is bounded by λ_t (the long-wave boundary) and λ_{cr}^1 for the first waveguide mode, $\lambda_b \approx 1.84a_s/\sqrt{\varepsilon_2}$, constituting about two octaves. There are no diffraction losses for the H_{00} wave and the losses in the dielectric are defined by

$$h''_\varepsilon = \frac{4.35\varepsilon k^2 \tan\delta}{\nu_1 h'} \text{ dB} \times \text{m}^{-2},$$

where $\tan\delta$ is the dielectric loss coefficient. The wave resistance of the cylindric slot line is $Z_{\text{csl}} = 120\pi^2 k/\eta h'$ ohm.

At $(\lambda_{\text{cr}} - \lambda_t)/\lambda_{\text{cr}} = 1 - \sqrt{1 - \varepsilon_{\text{ef}}^{-1}}$, a situation arises in which $v_\Phi > c/\varepsilon_2$. This gives rise to the Cerenkov radiation of the slot wave from the line at an angle whose cosine is

$$\cos\alpha = c/(v\sqrt{\varepsilon_2}) = h'/k.$$

For a cylindric slot line with metal walls of a finite thickness, we have

$$h' = k\left[1 - \frac{1}{2\pi(ka_s)^2}\left(Q_s + \ln\frac{1}{\sin 0.5\theta}\right)^{-1}\right]^{0.5},$$
$$h'' = 0.54\,\frac{\pi}{h'}(h'^2 - k^2)\left(\ln\frac{1}{\sin 0.5\theta} - Q_s\right) \text{ dB} \times \text{m}^{-1}, \tag{8}$$

where h'' describes the diffraction losses. The critical wavelength $\lambda_{\text{cr}} = 2\pi a_s[Q_s + 2\pi\ln(1/\sin 0.5\theta)]$. It also follows from (8) that the quasi-H_{00} wave is rapid and its phase velocity $v_\Phi = ck/h'$. Consequently, the propagation of rapid waves in a cylindric slot line is accompanied by their radiation into the free space, which forms a regular Cerenkov cone with an aperture angle 2α. The directivity pattern was found by the Kirchhoff method to be [2, 3]

$$|\psi(\gamma)|^2 = \frac{h''^2}{h''^2 + \zeta^2}\left(\cos^2\frac{L\zeta}{2} + \sin^2\frac{L\zeta}{2}\coth^2\frac{Lh''}{2}\right),$$

where $\zeta = h' - k\cos\gamma$; γ is the angle of observation in the line axis plane. It must be noted that a complex transition process occurs near the cylindric slot line, which does not conform to the Huygens principle at certain distances from the structure in the transverse direction. It turns out [3, 13] that the total field, existing across the entire space and being radiated from the cylindric slot line, includes the field of the outgoing wave and the radiation field. The former field exists near the line and falls sharply at infinity. The latter one, on the contrary, is much smaller near the line than the outgoing wave, but it is this field that becomes dominating with growing transverse coordinate. Therefore they cannot be combined into a single field, because their properties are different: the outgoing wave propagates at the velocity $v_\Phi > c$, and in the line's vicinity its wavelength can be measured as $\lambda = \lambda_g = 2\pi/h'$. Away from the line, where the total field is determined by the radiation field, the quantity h' has the formal meaning of the projection of the wave vector $\mathbf{k}$, directed at angle α to the Oz axis, onto that axis; the radiation field propagates at velocity c, therefore the value of λ_g cannot be measured away from the line.

Mirror slot lines are a modification of cylindric slot lines. In such a line, a portion of the metal screen is made in the form of a conducting substrate and the slot is formed by the shield edge and the substrate. The cylindric rod inside the mirror slot line may have virtually any shape. The simplest mirror slot line has a square cross section. Due to its substrate, such a line has high operational characteristics and is sufficiently rigid. It can be used as a basis for manufacturing various integral components and functional units. Various structural features can be arranged under the substrate. The dispersion relation for a mirror slot line is obtained from (6) for

$$\eta = \frac{\pi d_2}{d_1} + 0.693 - \ln\left[1 - \left(1 - \frac{\pi d_1^2}{2S_{\text{csl}}}\right)^{0.5}\right]$$

with appropriate values of ε_{ef}. An analysis of these relations shows that the widest pass band corresponds to a mirror slot line with a square cross section, with $2a = b$, so that the section area $S_{\text{cs}} = 2a^2$; the pass band is 2 to 3 octaves, enabling the entire mm band to be covered by a single line. When comparing the mirror slot line with the strip slot line and the screened slot line, or the fin-line, it must be pointed out that the mirror slot line has smaller dimensions, a higher field concentration in the slot, and a higher degree of screening and noise resistance. Compared with the mirror slot line, the fin-line has a fourfold greater size and a 40 to 50% narrower bandwidth; the fin-line wave resistance is the same as that of the cylindric slot line and twice as high as that of the mirror slot line.

EXPERIMENT

Experimental investigation of OW is performed by both conventional and specially developed techniques. Slot lines offer possibilities for designing new methods to measure the field structure, the dispersion, the losses, the OW mode composition, as well as new measuring devices on their basis. An integrated technique for experimental investigation of OW should allow studying not only the slot wave but also the Sommerfeld wave, as well as other wave types, which is associated with measuring various phase velocities along the OW. The existence of different wave types in OW can also be detected from the pattern of longitudinal wave distribution in the traveling-wave mode. For this purpose a special measuring line was designed, whose principle of operation is based on a mirror slot line [2, 3] and which allows one to study the dispersion, the thermal losses, the wave resistances, etc. in a wide frequency band. Moreover, this line allows one to produce three-dimensional OW field distributions and to study the signal propagation through a branching multichannel integral circuit (the measurements are performed by the same detector, with no switchings in the circuit). A two-coordinate measuring line permits an integrated quality monitoring of integral circuits, including the detection of local nonuniformities.

Systematic experiments were carried out with the setups developed on the basis of cylindric and mirror slot lines. The calculated retardation factors were found to coincide with the experimental ones. The same holds for the slot-wave dispersion and its losses. For instance, for a copper cylindric slot line ($2a_s = 1.35$ mm, $\theta = 33°$) the experimental losses at $\lambda_0 = 4.1$ mm are about 5 dB/m (the calculated value is about 6 dB/m); the loss calculation accuracy becomes better as the slot width is reduced. These experiments show that such a line may well be employed as a miniature transmission line in the mm band.

We also studied [3, 13] the mode composition of a cylindric slot line, using the phenomena of wave interference in such structures and diffraction analysis. These enable one to clearly separate the wave types (the slot waves, the Sommerfeld waves, and the dielectric waveguide waves) and create the most favorable conditions for the propagation of the slot waves. Based on these experiments, we were able to propose such line designs in which the mm-band attenuation is reduced to 0.2 dB/m.

An important issue in the OW research is the investigation of the outgoing waves, which, in contrast to the slot (i. e., surface) waves, under certain conditions can leave the OW and go to the free space in the form of bulk waves. The formation of an outgoing wave and its separation from the OW occur in the near zone, so it is in that zone that the greatest amount of information about these waves' properties can be obtained. The propagation of rapid waves in a cylindric slot line is accompanied by their radiation at the Cerenkov angle $\alpha = \cos^{-1}(h'/k)$. If the internal volume is filled by a dielectric, the rapid outgoing wave becomes a slow surface wave and the OW radiation ceases. All stages of the outgoing wave field transformation can be traced experimentally (by the amplitude distribution of the electric field component): at the input end of the OW, the field is concentrated at the slot; then, at a certain distance, it separates at the Cerenkov angle; finally, at some larger distances from the source, radiation arises at an angle $\gamma_r > \alpha$ which is different from the Cerenkov radiation. Consequently, a complicated wave process (a catastrophic phenomenon) occurs near the OW (at a distance greater than $2\lambda_0$), which is probably associated with the Morse critical points of the dispersion equation. The wave which goes out at the angle γ_r can be regarded as an independently excited mode.

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