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CASCADE BUNCHING OF ELECTRONS IN AN A. C. FIELD

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A discrete longitudinal interaction of an electron beam with a nonsynchronous polyharmonic electromagnetic field has been considered at arbitrary angles of transit and a spatial separation of regions of modulation with different frequencies. A one-dimensional theory has been constructed for cascade modulation and bunching of electrons in a flow passing through a sequence of adjoining electromagnetic gaps of arbitrary lengths in which a. c. electgromagnetic fields with various frequencies, phases, and forms of amplitude distribution were excited. In the approximation linear in field, with allowance for the Coulomb forces of the electron volume charge, expressions are obtained for the particle flight time and for the output current spectrum. Passages to the limits of special cases of modulators of a multiresonator klystron and a cascade monotron are considered.

INTRODUCTION

Averaged motion of a charged particle in a varying field of a weakly inhomogeneous electromagnetic wave has a quasipotential character [1, 2] and does not depend on the quick-oscillating phase of vibrations. A breakdown of an adiabatically smooth variation of the field amplitude over some region of space leads to the appearance of a dependence of the particle "slow" velocity on the phase of flight through this region [3–9]. Subsequent formation of electron bunches in the a. c. guiding field is a result of the modulation of the mean velocity. If the electrons move through several regions of perturbation of a weakly inhomogeneous wave, a process develops in their flow which is quite analogous to a cascade bunching of the electron beam in a multiresonator klystron.

This paper presents the results of a theoretical analysis of a more general process of consecutive modulation and cascade bunching of an electron flow that has passed through a series of adjacent regions of arbitrary length. In each region, a weakly inhomogeneous a. c. field of an arbitrary frequency with different spatial distribution of the amplitude is excited. It is assumed that the spatial homogeneity of the field is broken only at the boundaries of these regions.

1. STATING OF THE PROBLEM

Let an electron flow get into a sequence (Fig. 1) of modulating gaps separated by grids or diaphragms which are transparent to the particles. Such an electrodynamic system can be obtained, e. g., by connecting in series a number of open or volume resonators and/or waveguides. Suppose that longitudinal a. c. electric fields of the form ${}^iE(x)\sin({}^i\omega t + {}^i\varphi_0)$, $i = 0, 1, \ldots, N$ with arbitrary frequencies ${}^i\omega$, initial phases ${}^i\varphi_0$, and amplitudes ${}^iE$ (where N is the number of separating grids) are excited in the gaps. (Here and in what follows the following notation system is adopted: the left superscript of a variable stands for the number of the electromagnetic gap the variable refers to, and the right subscript stands for the number of the separating diaphragm or grid).

Let us also suppose that the field amplitudes in the gaps vary adiabatically slow (upon variation of the coordinates), and the diaphragms with holes or the grids between them are impermeable to the alternating

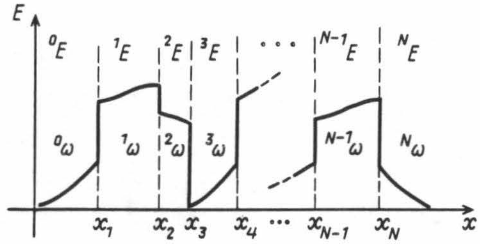

Fig. 1

Distribution of the amplitude of a longitudinal a. c. electric field along the electron flow.

field and transparent to electrons. Furthermore, we shall disregard the transverse structure of the a. c. field, i. e., we shall assume the electron flow to be sufficiently thin.

Let us first consider the interaction of electrons and the field in the kinematic approximation. In this case the evolution of the mean velocity of the electron motion in the ith intergrid gap (${}^ix \in (x_i, x_{i+1})$) has a quasipotential character and is described by the adiabatic invariant [1, 2]:

$$ {}^i\overline{\psi}^2 + (1/2)\,{}^i\varepsilon^2({}^i\chi) = \text{const}, \tag{1} $$

where ${}^i\overline{\psi} \equiv {}^i\overline{v}/v_0$; v_0 is the speed of the electron at the entrance to the modulator; ${}^i\varepsilon \equiv (e_0/m_0)\,{}^iE/({}^i\omega v_0)$, ${}^i\chi \equiv {}^i\omega\,{}^ix/v_0$ is the normalized amplitude of the field and the coordinate, respectively; l_0, m_0 are the electron charge and mass, respectively. Consideration for the conservation law (1) is especially important in the analysis of the nonlinear energy exchange between the formed electron bunch and the a. c. field of a large amplitude in the exit gap of the electronic device.

2. MODULATION OF THE ELECTRON MEAN VELOCITY

From the condition of continuity of the kinetic energy of an electron during its transit through the interface between the adjoining gaps that is nontransparent for the field and considering the weak spatial inhomogeneity of the wave amplitude to the left and to the right of the grid, we immediately obtain the law of modulation of the "slow" particle speed in the following form

$$ \overline{\psi_i^+} = \overline{\psi_i^-} + \varepsilon_i^+ \cos({}^i\omega t_i + {}^i\varphi_0) - \varepsilon_i^- \cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0), \tag{2} $$

where $\overline{\psi_i^-}$, ε_i^- and $\overline{\psi_i^+}$, ε_i^+ are the normalized mean speed and the normalized amplitude of the field to the left and to the right of the ith step, respectively. Recall that in the adopted notation we have $\varepsilon_i^+ \equiv {}^i\varepsilon_i$, $\varepsilon_i^- \equiv {}^{i-1}\varepsilon_i$, $\overline{\psi_i^+} \equiv {}^i\overline{\psi}_i$, $\overline{\psi_i^-} \equiv {}^{i-1}\overline{\psi}_i$.

In the study of processes of modulation and bunching (but not of taking away the energy) of electrons we shall hereafter assume that the amplitude of the guiding field is fairly small:

$$ {}^i\varepsilon \ll {}^i\overline{\psi}, \tag{3} $$

so that term quadratic in field in (1) can be neglected, so the angle ${}^n\theta$ and the time of flight ${}^n\tau$, as well as the relation between the input (${}^n\overline{\psi}_n$) and the output (${}^n\overline{\psi}_{n+1}$) mean electron velocities in the gap, can be approximately written as

$$ {}^n\theta = {}^n\theta_0/{}^n\overline{\psi}_n, \quad {}^n\tau = {}^nd/(v_0\,{}^n\overline{\psi}_n)), \quad {}^n\overline{\psi}_{n+1} = {}^n\overline{\psi}_n, \tag{4} $$

where ${}^n\theta_0$ is the unperturbed angle of flight in the nth gap of thickness nd. Thus, in the linear (in field) approximation we are neglecting the influence of the longitudinal spatial distribution of the field amplitude on the angle and time of electron flight in the gap and on the magnitude of its mean velocity.

Using Eqs. (4) and the law of modulation (2) we shall write the chain of equalities

$$\begin{aligned}
{}^{n}\overline{\psi}_n &= {}^{n-1}\overline{\psi}_{n-1} + {}^{n}\varepsilon_n \cos({}^{n}\omega t_n + {}^{n}\varphi_0) - {}^{n-1}\varepsilon_n \cos({}^{n-1}\omega t_n + {}^{n-1}\varphi_0) \\
&= {}^{n-2}\overline{\psi}_{n-2} + {}^{n-1}\varepsilon_{n-1} \cos({}^{n-1}\omega t_{n-1} + {}^{n-1}\varphi_0) - {}^{n-2}\varepsilon_{n-1} \cos({}^{n-2}\omega t_{n-1} + {}^{n-2}\varphi_0) \\
&\quad + {}^{n}\varepsilon_n \cos({}^{n}\omega t_n + {}^{n}\varphi_0) - {}^{n-1}\varepsilon_n \cos({}^{n-1}\omega t_n + {}^{n-1}\varphi_0) = \ldots \\
\ldots &= {}^{0}\overline{\psi}_1 + {}^{1}\varepsilon_1 \cos({}^{1}\omega t_1 + {}^{1}\varphi_0) - {}^{0}\varepsilon_1 \cos({}^{0}\omega t_1 + {}^{0}\varphi_0) + \ldots \\
&\quad \cdots + {}^{n}\varepsilon_n \cos({}^{n}\omega t_n + {}^{n}\varphi_0) - {}^{n-1}\varepsilon_n \cos({}^{n-1}\omega t_n + {}^{n-1}\varphi_0),
\end{aligned}$$

where ${}^{0}\overline{\psi}_1 = 1$ is the speed at the entrance to the modulator. As a result we have

$$ {}^{n}\overline{\psi}_n = 1 + \sum_{i=1}^{n} \left[{}^{i}\varepsilon_i \cos({}^{i}\omega t_i + {}^{i}\varphi_0) - {}^{i-1}\varepsilon_i \cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0) \right]. \tag{5}$$

One can see from this expression that in the linear approximation the mean speed at the entrance to the nth gap includes, in the additive way, the results of modulation at each preceding grid or diaphragm. The a. c. fields to the left and to the right of the separating diaphragm, that produce a step-like amplitude, give an independent contribution to this process.

3. TOTAL TIME OF FLIGHT

Substitution of (5) into (4) with regard to (3) gives the following expression for the time of particle motion in the nth gap:

$$ {}^{n}\tau = \frac{{}^{n}d}{v_0} \left\{ 1 + \sum_{i=1}^{n} \left[{}^{i-1}\varepsilon_i \cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0) - {}^{i}\varepsilon_i \cos({}^{i}\omega t_i + {}^{i}\varphi_0) \right] \right\}.$$

Let us calculate the total time $T_{1N} \equiv \sum_{n=1}^{N-1} {}^{n}\tau$ of electron flight through the modulator:

$$ T_{1N} = \frac{L_{1N}}{v_0} + \frac{1}{v_0} \sum_{n=1}^{N-1} \sum_{i=1}^{n} {}^{n}d \left[{}^{i-1}\varepsilon_i \cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0) - {}^{i}\varepsilon_i \cos({}^{i}\omega t_i + {}^{i}\varphi_0) \right],$$

where L_{1N} is the length of the modulator. Using the relation

$$ \sum_{n=1}^{N-1} \sum_{i=1}^{n} a_{in} = \sum_{i=1}^{N-1} \sum_{n=i}^{N-1} a_{in}$$

we find from the last expression the relation between the time instants of getting into the modulator (t_1) and getting out of the last gap (t_N) for the particle:

$$ t_N = t_1 + \frac{L_{1N}}{v_0} - \sum_{i=1}^{N-1} \frac{L_{iN}}{v_0} \left[{}^{i}\varepsilon_i \cos({}^{i}\omega t_i + {}^{i}\varphi_0) - {}^{i-1}\varepsilon_i \cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0) \right],$$

here L_{iN} is the distance from the ith to the Mth grid.

We introduce the bunching parameters X_{iN}^{+} and X_{iN}^{-} characterizing the contribution of the independent influence of the a. c. field on each side of the ith modulating diaphragm to the formation of an electron bunch:

$$ X_{iN}^{+} \equiv \frac{{}^{i}E(x_i) L_{iN}}{2U_0}, \qquad X_{iN}^{-} \equiv \frac{{}^{i-1}E(x_i) L_{iN}}{2U_0}.$$

Then the dependence $t_N(t_1)$ in question takes the final form

$$t_N = t_1 + \frac{L_{1N}}{v_0} - \sum_{i=1}^{N-1}\left[\frac{X_{iN}^{+}}{{}^{i}\omega}\cos({}^{i}\omega t_i + {}^{i}\varphi_0) - \frac{X_{iN}^{-}}{{}^{i-1}\omega}\cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0)\right]. \tag{6}$$

4. SPECTRAL DENSITY OF THE OUTGOING CURRENT

From Eq. (6) and from the charge conservation law we shall find the spectral density $I(\omega)$ of the convection current at the exit of the modulator:

$$I(\omega) = \frac{I_0}{2\pi}\int_{-\infty}^{\infty}\exp\{-i\omega t_N(t_1)\}\,dt_1.$$

Omitting intermediate calculations typical of the theory of cascade bunching, we present here only the final expression for the spectrum of the current:

$$\begin{aligned}
I(\omega) = I_0 &\sum_{\{\{p\}\}=-\infty}^{\infty} J_{p_{N-1}^{+}}\left(\frac{\omega}{{}^{N-1}\omega}X_{N-1,N}^{+}\right) J_{p_{N-1}^{-}}\left(\frac{\omega}{{}^{N-2}\omega}X_{N-1,N}^{-}\right)\\
&\times J_{p_{N-2}^{+}}\left[\frac{\omega}{{}^{N-2}\omega}X_{N-2,N}^{+} - \left(p_{N-1}^{+}\frac{{}^{N-1}\omega}{{}^{N-2}\omega} - p_{N-1}^{-}\right)X_{N-2,N-1}^{+}\right]\\
&\times J_{p_{N-2}^{-}}\left[\frac{\omega}{{}^{N-3}\omega}X_{N-2,N}^{-} - \left(p_{N-1}^{+}\frac{{}^{N-1}\omega}{{}^{N-3}\omega} - p_{N-1}^{-}\frac{{}^{N-2}\omega}{{}^{N-3}\omega}\right)X_{N-2,N-1}^{-}\right]\\
&\cdots\times J_{p_1^{+}}\left[\frac{\omega}{{}^{1}\omega}X_{1N}^{+} - \left(p_{N-1}^{+}\frac{{}^{N-1}\omega}{{}^{1}\omega} - p_{N-1}^{-}\frac{{}^{N-2}\omega}{{}^{1}\omega}\right)X_{1,N-1}^{+} - \cdots - \left(p_2^{+}\frac{{}^{2}\omega}{{}^{1}\omega} - p_2^{-}\right)X_{12}^{+}\right]\\
&\times J_{p_1^{-}}\left[\frac{\omega}{{}^{0}\omega}X_{1N}^{-} - \left(p_{N-1}^{+}\frac{{}^{N-1}\omega}{{}^{0}\omega} - p_{N-1}^{-}\frac{{}^{N-2}\omega}{{}^{0}\omega}\right)X_{1,N-1}^{-} - \cdots - \left(p_2^{+}\frac{{}^{2}\omega}{{}^{0}\omega} - p_2^{-}\frac{{}^{1}\omega}{{}^{0}\omega}\right)X_{12}^{-}\right]\\
&\times\exp\Big\{-j\Big[p_{N-1}^{+}\left({}^{N-1}\omega T_{N-1,N} - {}^{N-1}\varphi_0 - \frac{\pi}{2}\right)\\
&- p_{N-1}^{-}\left({}^{N-2}\omega T_{N-1,N} - {}^{N-2}\varphi_0 - \frac{\pi}{2}\right) + \cdots + p_1^{+}\left({}^{1}\omega T_{1N} - {}^{1}\varphi_0 - \frac{\pi}{2}\right)\\
&- p_1^{-}\left({}^{0}\omega T_{1N} - {}^{0}\varphi_0 - \frac{\pi}{2}\right)\Big]\Big\}\delta\left[\omega - \sum_{i=1}^{N-1}(p_i^{+}\,{}^{i}\omega - p_i^{-}\,{}^{i-1}\omega)\right],
\end{aligned} \tag{7}$$

where J_p is the Bessel function of the pth order, T_{ij} is the time of an unperturbed electron motion from the ith to the jth diaphragm, δ is Dirac's delta-function, $\{\{p\}\} \equiv p_1^{+}, p_1^{-}, p_2^{+}, p_2^{-}, \ldots, p_{N-1}^{+}, p_{N-1}^{-}$.

Analysis of expressions (2), (6), and (7) shows that modulation of the electron velocity by an a. c. field at each interface between the interaction regions exerts independent influence on the spectral density of the bunched electron flow. This property of the interaction in question opens far broader opportunities for controlling the harmonic structure of the current and formation of an electron bunch than existing possibilities of the cascade klystron buncher.

Indeed, let a flight klystron contain $n-1$ modulating resonators, each of them in the general case having arbitrary frequency, phase, and amplitude of the a. c. field, and also a coordinate of its location. Then, the dimension of the space of the control parameters, in which the optimization of the bunching process is carried out, is equal to $4n-4$. In the case considered in our work, when the field amplitude has the same number of steps $(2n-2)$ as in the above-mentioned example of a klystron, the maximal number of degrees of freedom is equal to the number of steps (in view of the independence of the coordinates of their locations) multiplied by two (the independent modulation by the field on each side of the grid is taken into account) and by three (with regard to the independent frequency, amplitude, and phase of every guiding field). As a result the dimension of the configuration space of optimization equals $12n-12$, which is three times greater than in the klystron buncher.

5. CONSIDERATION FOR THE VOLUME CHARGE AND THE SAGGING OF THE FIELD

One can approximately evaluate the influence of sagging of the a. c. field into the apertures of the diaphragm on the process of electron bunching by introducing into expressions obtained the corresponding correcting multiplyers. These are the efficiencies of modulation of the mean velocity on a nonideal step of the field [8, 9] which have the form $a = \exp\{-l/\lambda_l\}$, where l is the effective depth of sagging of the field into the apertures for the electron transit. The introduction of these coefficients imposes substantial limitations on the field transparancy of the grids and diaphragms: the velocity modulation on the amplitude step and the subsequent formation of electron bunches are effective only when the decrease of the amplitude at the edge of the electromagnetic gap is sufficiently rapid: $l \lesssim \lambda_e$. This requirement is in fact one of the basic factors limiting the maximal working frequency of the interaction under consideration. Its estimate for various concrete examples, made in [9, 10], gave a value lying in the infrared frequency region.

Taking account of the electron volume charge will be carried out on the basis of the solution of Greenberg's evolution equation [11] which describes the average motion of charged particles in a weakly inhomogeneous electromagnetic wave in the presence of a stationary compensating ionic background:

$$\frac{d^{2n}\overline{\psi}}{d^n\varphi^2} + ({}^n\overline{\omega}_e^2 + {}^n\overline{\omega}_f^2)\,{}^n\overline{\psi} = {}^n\overline{\omega}_e^2, \tag{8}$$

where ${}^n\varphi \equiv {}^n\omega t$ is the current phase of the field, ${}^n\overline{\omega}_e^2$ is the square of the normalized electron plasma frequency, ${}^n\overline{\omega}_f^2$ is the correction to it. The correction arises on account of the action of the quasipotential forces of Gaponov and Miller and is proportional to the square of the amplitude of the a. c. field.

In the linear approximation we neglect the correction ${}^n\overline{\omega}_f^2$ and we find from (8) that the evolution of the slow particle velocity in the gap with a weakly inhomogeneous a. c. field is determined by the selfconsistent equation of oscillations

$$\frac{d^{2n}\overline{\psi}}{d^n\varphi^2} + {}^n\overline{\omega}_e^2({}^n\overline{\psi} - 1) = 0,$$

obtained in [12] for the one-dimensional drifting electron flow. A solution of this equation gives the relation (necessary for further calculations) of the velocities to their derivatives at the gap boundaries as well as the flight angle in it:

$${}^n\theta = {}^n\theta_0\left[1 - ({}^n\overline{\psi}_n - 1)\frac{\sin{}^n\overline{\omega}_e\,{}^n\theta_0}{{}^n\overline{\omega}_e\,{}^n\theta_0} - {}^n\overline{\psi}'_n\,\frac{1-\cos{}^n\overline{\omega}_e\,{}^n\theta_0}{{}^n\overline{\omega}_e^2\,{}^n\theta_0}\right],$$

$${}^n\overline{\psi}_{n+1} - 1 = ({}^n\overline{\psi}_n - 1)\cos{}^n\overline{\omega}_e\,{}^n\theta_0 + \frac{1}{{}^n\overline{\omega}_e}\,{}^n\overline{\psi}'_n\sin{}^n\overline{\omega}_e\,{}^n\theta_0,$$

$${}^n\overline{\psi}'_{n+1} = -{}^n\overline{\omega}_e({}^n\overline{\psi}_n - 1)\sin{}^n\overline{\omega}_e\,{}^n\theta_0 + {}^n\overline{\psi}'_n\cos{}^n\overline{\omega}_e\,{}^n\theta_0.$$

In order to calculate the total transit time of an electron in the buncher, in addition to the already written expressions we shall use expression (2) for the velocity modulation on the amplitude step, and we shall also assume that the time derivative of the slow velocity does not change on the particle passage from one gap to another:

$$\left.\frac{1}{{}^i\overline{\omega}_e}\frac{d^i\overline{\psi}}{d^i\varphi}\right|_{{}^i\varphi_i} = \left.\frac{1}{{}^{i-1}\overline{\omega}_e}\frac{d^{i-1}\overline{\psi}}{d^{i-1}\varphi}\right|_{{}^{i-1}\varphi_i}. \tag{9}$$

Since the evolution of the mean velocity and its derivatives in the gap are determined in the linear approximation only by the Coulomb forces of electrons and ions, expression (9) actually means equality of these forces on the left and on the right of the amplitude step and it can be treated as a transparency condition of the ith grid or of the diaphragm for waves of the electron volume charge.

Using the adopted approximations and taking account in a consistent way, on the basis of the derived expressions, of the effect of the preceding velocity modulation on the electron motion in the nth gap, we find the time ${}^n\tau$ of particle transit:

$${}^n\tau = \frac{{}^nd}{v_0}\left\{1 + M_{n0}\sum_{i=1}^{n}[{}^{i-1}\varepsilon_i\cos({}^{i-1}\omega t_i + {}^{i-1}\varphi_0) - {}^i\varepsilon_i\cos({}^i\omega t_i + {}^i\varphi_0)]\cos\left(\frac{{}^n\overline{\omega}_e\,{}^n\theta_0}{2} + \sum_{j=1}^{n-i}{}^{n-j}\overline{\omega}_e\,{}^{n-j}\theta_0\right)\right\}.$$

As a result of subsequent transformations (which, beginning from this stage, coincide with those mentioned in the kinematical section of the theory), for the dependence $t_N(t_1)$ and for the spectral density of the bunched flow $I(\omega)$ we obtain expressions that coincide with Eqs. (6) and (7) with the only distinction that the intergrid distances L_{ik} in them should be replaced by the quantities

$$L'_{ik} \equiv \sum_{n=i}^{k-1} {}^n d^n M_0 \cos\left(\frac{{}^n\overline{\omega}_e\, {}^n\theta_0}{2} + \sum_{j=1}^{n-i} {}^{n-j}\overline{\omega}_e\, {}^{n-j}\theta_0\right).$$

Here ${}^n M_0 \equiv \sin({}^n\overline{\omega}_e\, {}^n\theta_0/2)/({}^n\overline{\omega}_e\, {}^n\theta_0/2)$ is the parameter of the volume charge taking account of the pushing action of the Coulomb forces in the nth electromagnetic gap, and the expressions in parentheses reflect the interference of the volume charge waves excited in various regions of the modulator.

6. SOME EXAMPLES

Let us show that the processes of modulation and formation of electron bunches in the flow in both the klystron and the monotron are special cases of the interaction discussed in the present communication. Indeed, consider a multifrequency klystron bunching [13]. In the model constructed by us, the following values of the parameters for uneven N, ${}^iE(x_i) = {}^iE(x_{i+1}) = {}^iE$, correspond to this example. That is, the amplitude of an a. c. field is constant in the uneven gaps and their length is supposed to be small (${}^i\tau^i\omega \ll 1$). In the even gaps ${}^iE(x) = 0$ the field is absent in the drift regions. Substituting these values into Eq. (6) and carrying out passages to the corresponding limits, we obtain the relation of the instants of electron arrival at (t_1) and departure from (t_n) the modulator in the following form:

$$t_n = t_1 + \frac{l_{1n}}{v_0} - \sum_{j=1}^{n-1} \frac{1}{j\omega} X_{jn} \sin\left({}^j\omega t_j + {}^j\varphi_0 + \frac{1}{2}{}^j\theta_0\right), \tag{10}$$

where $n = (N-1)/2$ is the number of resonators in the klystron ($n-1$ of them are modulating resonators); $X_{jn} \equiv (1/2)^j M_0{}^j\xi\theta_{0jn}$ is the bunching parameter; ${}^jM_0 \equiv \sin({}^j\theta_0/2)/({}^j\theta_0/2)$ is the efficiency of modulation; ${}^j\theta_0 \equiv {}^j\omega^j d/v_0$ is the unperturbed transit angle in the jth resonator; $\xi_j \equiv {}^j\widetilde{U}/U_0$ is the coefficient of voltage utilization; ${}^j\widetilde{U}$ is the amplitude of the a. c. field in the jth resonator; $\theta_{0jn} \equiv {}^j\omega l_{jn}/v_0$ is the transit angle between the jth and nth gaps. Expression (10) completely coincides with the corresponding expression in [13].

On the other hand, using Graf's addition theorem for the Bessel functions [14], we obtain from (7) in the same approximations the expression for the spectral density of the convection current at the exit of the $(n-1)$ resonator buncher of the klystron:

$$\begin{aligned} I(\omega) = I_0 \sum_{\{p_i\}=-\infty}^{\infty} & J_{p_1}\left(\frac{\omega}{{}^{n-1}\omega} X_{n-1,n}\right) J_{p_2}\left(\frac{\omega}{{}^{n-2}\omega} X_{n-2,n} - p_1 \frac{{}^{n-1}\omega}{{}^{n-2}\omega} X_{n-2,n-1}\right) \\ & \times J_{p_3}\left(\frac{\omega}{{}^{n-3}\omega} X_{n-3,n} - p_1 \frac{{}^{n-1}\omega}{{}^{n-3}\omega} X_{n-3,n-1} - p_2 \frac{{}^{n-2}\omega}{{}^{n-3}\omega} X_{n-3,n-2}\right) \\ & \cdots \times J_{p_{n-1}}\left(\frac{\omega}{{}^1\omega} X_{1n} - p_1 \frac{{}^{n-1}\omega}{{}^1\omega} X_{1,n-1} - p_2 \frac{{}^{n-2}\omega}{{}^1\omega} X_{1,n-2}\right. \\ & \left. \cdots - p_{n-2} \frac{{}^2\omega}{{}^1\omega} X_{12}\right) \exp\left\{-j\left[p_1\left({}^{n-1}\omega T_{n-1,n} - {}^{n-1}\varphi_0 - \frac{1}{2}{}^{n-1}\theta_0\right)\right.\right. \\ & \left.\left. + p_2\left({}^{n-2}\omega T_{n-2,n} - {}^{n-2}\varphi_0 - \frac{1}{2}{}^{n-2}\theta_0\right) + \cdots + p_{n-1}\left({}^1\omega T_{1n} - {}^1\varphi_0 - \frac{1}{2}{}^1\theta_0\right)\right]\right\} \delta\left(\omega - \sum_{j=1}^{n-1} p_j\, {}^{n-j}\omega\right), \end{aligned} \tag{11}$$

which also coincides with the analogous expression in [13]. Since the derivation of the spectrum of the current (11) from expression (10) has been carried out earlier in [13], the commutative implicative diagram

$$\begin{array}{ccc} (6) & \longrightarrow & (10) \\ \downarrow & & \downarrow \\ (7) & \longrightarrow & (11) \end{array}$$

closes, and this proves that the cascade multifrequency klystron modulation and bunching are special cases of a more general method of guiding electron flow considered in the present paper.

For a single-frequency klystron buncher (${}^1\omega = {}^2\omega = \cdots = {}^{n-1}\omega = \omega$) with narrow gaps and synphase fields in them (${}^i\varphi_0 = {}^i\theta_0 = 0,\ i = 1, \ldots, n-1$) we obtain from (11) an expression for the amplitudes of the output current harmonics, which we shall use in the subsequent analysis:

$$\begin{aligned} I_m = I_0 \sum_{\{p_i\}=-\infty}^{\infty} & J_{p_1}(mX_{n-1,n}) J_{p_2}(mX_{n-2,n} - p_1 X_{n-2,n-1}) J_{p_3}(mX_{n-3,n} \\ & - p_1 X_{n-3,n-1} - p_2 X_{n-3,n-2}) \ldots J_{p_{n-1}}(mX_{1n} - p_1 X_{1,n-1} - p_2 X_{1,n-2} \\ & \cdots - p_{n-2} X_{12}) \exp\left[-j\omega(p_{n-1}T_{1n} + p_{n-2}T_{2n} + \cdots + p_1 T_{n-1,n})\right], \end{aligned} \tag{12}$$

here for each value of $m = 0, \mp 1, \mp 2, \ldots, p_i$ take only such values that $\sum_{i=1}^{n-1} p_i = m$.

Now we shall consider the interaction of the a. c. field with electrons in a classical monotron [15]. To this end we shall assume in our theoretical model that $N = 2$, ${}^0E = {}^2E = 0$, ${}^1E(x) = \text{const}$, ${}^1\varphi_0 = 0$. Then we obtain from (7)

$$I(\omega) = I_0 \sum_{p=-\infty}^{\infty} J_p\left(\frac{\omega}{{}^1\omega} X_{12}^{+}\right) \exp\left\{-jp\left({}^1\omega T_{12} - \frac{\pi}{2}\right)\right\} \delta(\omega - p\,{}^1\omega).$$

From here it follows that at the exit of the monotron the harmonic structure of the convection current has the following form:

$$i(t_N) = \int_{-\infty}^{\infty} I(\omega) \exp\{j\omega t_N\}\, d\omega = I_0 + 2I_0 \sum_{p=1}^{\infty} I_p\left(\frac{1}{2} p\xi\right) \cos\left[p\left({}^1\omega t_N + \frac{\pi}{2} - {}^1\theta_0\right)\right],$$

where $\xi \equiv {}^1\widetilde{U}/U_0$ is the ratio of the a. c. field voltage in the gap to the flow potential. This expression corresponds to the spectrum of the output current given in [16]. Thus, modulation and bunching of electrons in the a. c. field of a classical monotron are also parts of the diverse electron-wave interactions considered in the present paper.

7. CASCADE MONOTRON OR ANTIKLYSTRON

By analogy with the klystron multiresonator buncher it is quite natural to try to describe cascade modulation and clustering in a monotron, when mean electron velocities are successively modulated by field amplitude steps in several space regions and at the same time the flow of particles is clustered in bunches in the same a. c. field of the resonator. Consider this process in more detail in the particular case of single-frequency interaction using the example of the electrodynamic system depicted in Fig. 2.

The electron flow 1 ejected by the gun 2 enters an open resonator formed by the spherical mirrors 3. A row of modulating metallic diaphragms 4 with apertures for electrons is placed across the way of the particle flow 1 in the field of the resonator. The polarization of the resonance wave is such that the vector of the electric field strength is parallel to the electron beam 1 and is perpendicular to diaphragms 4. The electron bunch, formed after passing $n-1$ modulating diaphragms, gets into the region of a strong a. c. field and, having transferred energy to it under properly selected phase conditions, hits the last diaphragm 5 that acts as a collector and forms the exit step of the field amplitude.

The distinction of the device in question from a multiresonator klystron is in particular in the fact that in the monotron everything is reversed: the role of the bunching space is played by the interdiaphragm region filled with an a. c. field, and the function of modulators of (mean) velocities is fulfilled by sections of diaphragms with apertures in which there is no a. c. field or it is markedly weakened. In this sence the device described is an antipode of a multiresonator klystron. It can be conditionally called a complementary cascade monotron or even antiklystron, because when the complementary fields of a klystron and a monotron (see

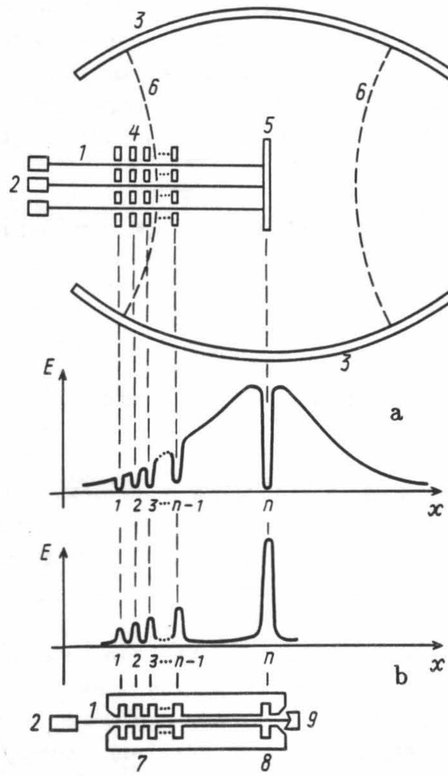

Fig. 2

Mutually complementary cascade monotron (a) and multiresonator klystron (b) and the respective longitudinal distributions of the a. c. field amplitude: (1) electron flows; (2) source of electrons; (3) mirrors of the open resonator; (4) modulating diaphragms; (5) exit diaphragm; (6) caustics of the resonance field; (7) modulating volume resonators; (8) exit resonator; (9) collector of waste electrons.

Fig. 2) are put together, a weakly inhomogeneous field of an open resonator is obtained whose effectiveness of interaction with the electron flow is equal to zero.

Further on we shall assume that the diaphragms are thin enough and one can neglect the distributions of the amplitudes on the left and on the right of them, and that the apertures in the diaphragms are small, so the a. c. field cannot practically penetrate into them. Under these simplifying approximations, after some transformations analogous to calculations in the case of a klystron, we obtain from expression (7) that the harmonic structure of the convection current of a bunched electron flow near the exit diaphragm 5 has the following form:

$$\begin{aligned} I_m = I_0 \sum_{\{q_i\}=-\infty}^{\infty} & J_{q_{n-1}}(mX_{n-1,n}) J_{q_{n-2}}(mX_{n-2,n} + q_{n-1}X_{n-2,n-1}) \\ & \cdots \times J_{q_1}(mX_{1n} + q_{n-1}X_{1,n-1} + q_{n-2}X_{1,n-2} + \cdots + q_2X_{12}) \\ & \times \exp\{j\omega(q_1T_{1n} + q_2T_{2n} + \cdots + q_{n-1}T_{n-1,n})\}, \end{aligned} \tag{13}$$

where the summation indices are such that $\sum_{i=1}^{n-1} q_i = m$, $X_{ij} \equiv \frac{1}{2}\,{}^i\xi^i M_0 \theta_{0ij}$ is the bunching parameter in the gap between the ith and jth diaphragms, ${}^i\xi$ is the coefficient of voltage utilization on the ith diaphragm equal to $E_i{}^i d/U_0$, E_i is the amplitude of the field strength near the ith diaphragm, ${}^i d$ is its thickness, ${}^i M_0 \equiv \sin({}^i\theta_0/2)/({}^i\theta_0/2)$, ${}^i\theta_0 \equiv \omega{}^i d/v_0$, T_{ij} and θ_{0ij} are the time and the angle of an unperturbed transit from the ith to the jth diaphragm.

Comparing the spectra of the current (12) with (13), one can see that both expressions actually coincide with each other with an accuracy up to renaming the summation indices ($q_{n-i} = -p_i$). This means that in the linear approximation the bunchers of the mutually complementing cascade monotron and multiresonator klystron provide an identical degree of compression and the form of the electron bunch.

Finally, using the theory (developed in [10, 17]) of energy extraction from a modulated electron flow by the field of an open resonator with an amplitude step, we calculate the electron efficiency η_n:

$$\eta_n = \mathrm{Re}\left(2\varepsilon_n\sqrt{1-\frac{1}{2}\varepsilon_n^2 i_1 - \frac{1}{2}\varepsilon_n^2 i_2}\right)$$

and the starting current I_{start} of the cascade monotron:

$$I_{\text{start}} = \left(U_0 \varkappa_n^2 \sum_{i=1}^{n-1} \gamma_i \theta_{0in} \cos\theta_{0in}\right)^{-1}.$$

Here $\varepsilon_n \equiv (e_0/m_0)E_n/(\omega v_0)$ is the amplitude of the a.c. field strength at the exit diaphragm; i_1, i_2 are the amplitudes of the +1st and +2nd harmonics of the convection current normalized to I_0 and determined from (13); $\varkappa_n$ is the electrodynamic parameter proportional to the quality of the resonator and relating the amplitude ε_n with the power P of the exciting signal: $\varepsilon_n = \varkappa_n\sqrt{P}$, γ_i is the ratio of the normalized field amplitude at the ith diaphragm to ε_n.

Analysis of the expressions for η_n and I_{start} shows that in a cascade monotron the energy is extracted from the 1st and the 2nd harmonics of the flow convection current. The possibilities of the cascade formation of an optimal bunch and of thereby raising the efficiency of the electron flow are then similar to the case of the n-resonator klystron. Moreover, it is possible to obtain a certain rise (by 5–8%) of the efficiency of energy extraction from the flow as a result of a general deceleration of electrons in an adiabatically slowly increasing a. c. field at the exit diaphragm 5 (see Fig. 2). Besides, if the modulating diaphragms in the cascade monotron are correctly located, a substantial lowering of the starting generation current is achieved. For example, for $\gamma_i = \gamma_1$, $\theta_{0in} \approx \theta_{01n}$, and $\cos\theta_{0in} \approx 1$, $i = 1, \ldots, n-1$ it follows from the last expression that I_{start} decreases $n-1$ times as compared to a single-gap monotron [18].

CONCLUSION

The process of longitudinal interaction of a multigap multifrequency a. c. electric field with an electron flow considered in the present paper includes the classical monotron and cascade klystron modulation and bunching of electrons, as well as analogous processes in a cascade monotron that have not been considered earlier. For this reason the described discrete interaction of electrons and the field can be treated as a unification or generalization of the klystron and monotron mechanisms of modulation, bunching, and extraction of energy. Here the "discreteness" of flow guiding should be understood as the leading role in it of comparatively small (of the order of λ_e) regions of fairly sharp spatial variation of the field, in which the adiabatic character of the longitudinal distribution of the amplitude is violated. In this case, in contrast to a prolonged interaction of the types of traveling-wave tube, backward-wave tube, diffraction-radiation generator, etc., the entire picture of electron bunching is produced on intergap grids or diaphragms with apertures and in the linear approximation does not depend on the presence and on the form of distribution of the a. c. field amplitude in the modulating gaps themselves.

The considered cascade bunching of electrons in a flow in an a. c. field seems to be of particular interest in electronic devices with millimeter or shorter wavelengths. In these wave bands it is rather difficult to achieve a classical klystron electrodynamic system in which the narrow modulating gaps alternate with regions of electron drift and a field structure of the opposite character is more natural: extensive (in the sense that flight angles are large), regions filled with an a. c. field of an open resonance structure alternate with small regions of phase modulation of electron flow. The latter regions are formed by sufficiently thin metallic or dielectric diaphragms that do not perturb too much the field of the structure as a whole. Besides, this arrangement of the electrodynamic system provides a relatively easy utilization of broad or multibeam electron flows, thereby increasing the power of the output signal of the created electronic devices.

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