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POLARIZATION OF COSMIC MICROWAVE BACKGROUND RADIATION

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The formation of the cosmic microwave background radiation polarization in observations of standard recombination is discussed. The polarization level in observations using antennas with different sizes of directivity patterns over the angle range varying from 3 to 15°C has been calculated. The formation of polarization was calculated in models with different kinds of spectra (for the spectrum index $n = 1$ and 1.5) and with different kinds of disturbances. Scalar disturbances and gravitational waves have been considered separately. The degree of the cosmic microwave background radiation polarization has been calculated.

1. INTRODUCTION

In recent years the discovery of large- and medium-scale anisotropy of the cosmic microwave background radiation (the CMBR) has been reported [1-7]. Density fluctuations are the most probable source of anisotropy (see, e.g., [8–10]). Gravitational waves (GWs) [11–13] which can also cause RR anisotropy are regarded as "minor additives" to density fluctuations [14–16]. This opinion is widespread because most of the theoretical models which describe the origination of density fluctuations and gravitational waves based on the inflation theory predict small contribution of GWs to the anisotropy as compared with density fluctuations. Such a hypothesis seems to be very attractive and reliable.

Nevertheless, the comparison of two-year observations in the "COBE" and "Tenerife" experiments has given somewhat unexpected results [4, 7]. These observations taken together lead to the spectral index of primary fluctuations, which is closer to $n = 1.5$ than to $n = 1$ predicted by the standard inflation theory. The question now arises of whether the standard inflation theory and the assumption that the influence of GWs is small are valid.

No doubt, the most correct hypothesis of the sources of this anisotropy will be the assumption that the main part of it owes its origin to primary scalar disturbances, whereas its other part to primary GWs. At present the contributions of these two primary types of disturbances cannot be separated. It will be made possible after measuring the polarization of the cosmic microwave background radiation [17–21].

The measurement of the cosmic microwave background radiation polarization is a convincing method for separating the contribution of density fluctuations to the anisotropy from the contribution of GWs.

In this paper we describe the main idea of the experiment proposed some years ago [22] and make some estimates of the expected signal for this experiment under different assumptions concerning the anisotropy sources. Namely, we estimate the degree of polarization for two spectrum indices $n = 1$ and $n = 1.5$, assuming that in the first case the anisotropy of the cosmic microwave background radiation is only caused by scalar disturbances and only by gravitational waves in the second case. These estimates provide some limiting values of the expected signal.

2. BASIC EQUATIONS

We assume Friedmann's model of the Universe with $\Omega_0 = 1$ and $\Lambda = 0$ and postulate the dust equation of state $p = 0$.

Considering metric fluctuations, we have the following expression for a linear interval $ds^2 = a^2(\eta)(d\eta^2 - (\delta_{\alpha\beta} + h_{\alpha\beta})dx^\alpha dx^\beta)$, where a small value $h_{\alpha\beta}$ meets Einstein's equation. It can be shown that this value may be determined by three independent types of quantities, scalar, vector, and tensor disturbances. In the present paper we shall consider the contribution to RR made by the quantities of the first and third types only; we shall consider in more detail the growing modes of adiabatic disturbances (scalar disturbances) and of cosmological gravitational waves (GWs).

We shall introduce Fourier transformation of metric disturbances in the conventional form. The scale chosen for scalar disturbances will be

$$h_{\alpha\beta}(k, \eta) = h_A(k)(k\eta)^2\gamma_\alpha\gamma_\beta$$

and for GWs

$$h_{\alpha\beta}(k, \eta) = (h_+ t_{\alpha\beta} + h_\times s_{\alpha\beta}) \cdot \frac{J_{3/2}(k\eta)}{(k\eta)^{3/2}},$$

where $h_A(k)$ are the stochastic variations possessing the property of being δ-correlated with the power spectrum

$$\langle h_A(k) h_A^*(q)\rangle = \frac{P(k)}{k^3}\delta(k - q)$$

and $P(k) = P_0 k^{n-1}$. The same equation holds for GWs as well

$$\langle h_+(k) h_+^*(q)\rangle = \frac{P_+(k)}{k^3}\delta(k - q),$$

$$\langle h_\times(k) h_\times^*(k)\rangle = \frac{P_\times(k)}{k^3}\delta(k - q).$$

The subscripts + and $\times$ correspond to each of the GW polarization states.

To describe the RR polarization, as in [23], we can introduce a symbolic vector $\hat{n}$. The quantity $\hat{n}$ is a function of kinetic distribution, which depends on the coordinates x_α and on the photon momenta p_α, and has the components n_l, n_r, and n_u. The components n_l and n_r are the occupation numbers of photons with perpendicular polarization planes. The quantity n_u describes the correlation between n_l and n_r. The total occupation number of photons is $n = n_l + n_r$. The anisotropy of the cosmic microwave background radiation is determined by fluctuations of this quantity. We denote these fluctuations by the vector $\hat{\delta}$. We represent the vector $\hat{\delta}$ using its conventional Fourier transform, and this will give us the kinetic equation for δ in the form

$$\frac{\partial\delta}{\partial\eta} + \frac{\partial\delta}{\partial x^\alpha}\cdot e^\alpha = \frac{1}{2}\cdot\frac{\partial h_{\alpha\beta}}{\partial\eta}e^\alpha e^\beta - \sigma_T N_e a(\eta)\left(\delta - \oint P(\Omega, \Omega')\delta(\Omega)\,d\Omega'\right), \tag{1}$$

where e^α is the unit vector. This equation holds true for the Rayleigh-Jeans part of the spectrum.

The expression for the scattering matrix P in terms of the angles θ and φ and particular terms of this equation can be found in [23, 24].

3. SOLUTION FOR PLANE WAVES

As will be shown in what follows, in order to find the anisotropy and polarization of the cosmic microwave background radiation for an arbitrary spectrum disturbance, it will suffice to solve equation (1) for the case of plane waves. Furthermore, it will make clear the physical meaning of the calculations.

We choose the coordinate system so that the disturbances should propagate along the z-axis. The cosine of the angle between the axis of propagation of photons and the z-axis will be denoted by μ. Then we have the following expression for the first term in the right-hand side of the equation

$$F_{GW} = (h_+\cos(2\varphi) + h_\times\sin(2\varphi))\frac{J_{5/2}(k\eta)}{(k\eta)^{3/2}}(1 - \mu^2), \tag{2}$$

$$F_{SP} = h_A k^2\eta(\mu^2 - 1/3), \tag{3}$$

where h_+ and $h_\times$ are the constant amplitudes of GWs, and h_A is the constant amplitude of scalar disturbances.

Here the monopole component of the anisotropy is excluded in order to obtain the Legendre polynomial of the second kind. For convenience, we choose the following form of δ

$$\delta = \alpha_{SP} \cdot (\mu^2 - 1/3) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha_+ \cos(2\varphi) \cdot (1-\mu^2) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$
$$+ \beta_{SP} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} (1-\mu^2) + \beta_+ \begin{pmatrix} (1+\mu^2)\cos(2\varphi) \\ -(1+\mu^2)\cos(2\varphi) \\ -4\mu\sin(2\varphi) \end{pmatrix}.$$

Here the subscript $\times$ is omitted, because the expressions for the equation with subscript $\times$ are similar to the equations with subscript $+$. Therefore, we only present the calculations for one degree of GW polarization.

After the substitution of the vector δ into inverted equation (1), we come to a system of integro-differential equations.

The polarization of the cosmic microwave background radiation arises mainly in the process of recombination. An optical thickness during that period is [25]

$$\tau(\eta) = \int_\eta^\infty \sigma_T N_e a(\eta')\, d\eta'.$$

This expression can be represented as

$$\tau = A\eta^{-3} \exp(-2\eta^2/\Delta\eta_r),$$

where $A = 7 \times 10^5 \times \Delta\eta_r = 0.1$, and the time of recombination end $\eta_r = 1$ corresponds to the red shift $z = 900$; $\Delta\eta_r$ is the interval in which the optical thickness for the Thomson scattering varied from 1 to 0.

We solve the system of integro-differential equations for $\eta > \eta_r$ under the assumption that $\alpha \gg \mu k\alpha$ and $\beta \gg \mu k\beta$. It is easy to see that such conditions are equivalent to the condition $k\Delta\eta_r \ll 1$.

This means that the functions α and β depend only on k and η, and we can represent our integro-differential system in the following manner:

$$\frac{d\alpha}{d\eta} = F - \frac{9}{10}\sigma_T N_e a(\eta)\alpha - \frac{6}{10}\sigma_T N_e a(\eta)\beta, \tag{4}$$
$$\frac{d\beta}{d\eta} = -\frac{1}{10}\sigma_T N_e a(\eta)\alpha - \frac{4}{10}\sigma_T N_e a(\eta)\beta. \tag{5}$$

The expressions for gravitational waves and for scalar disturbances are identical. Introducing the function $\xi = \alpha + \beta$ [18] and summing up these equations, we obtain a simple system of differential equations. An analytical solution of this system of equations has the form

$$\alpha = \frac{1}{7}\int_0^\eta F(\eta')(6 \cdot \exp(-\tau(\eta,\eta')) + \exp\left(-\frac{3}{10}\tau(\eta,\eta')\right) d\eta', \tag{6}$$

$$\beta = \frac{1}{7}\int_0^\eta F(\eta')\left(\exp(-\tau(\eta,\eta')) - \exp\left(-\frac{3}{10}\tau(\eta,\eta')\right)\right) d\eta', \tag{7}$$

where τ is the optical thickness from η to η'.

The expression for the function $F(k, \eta)$ will be presented in what follows both for gravitational waves and for scalar disturbances as an expansion into a Taylor series. Solving equations (7) and (8) numerically, we can find α and β. The solution shows that for scalar disturbances and gravitational waves their contributions to the anisotropy and to the degree of polarization will differ.

We have calculated the degree of polarization and the anisotropy of the cosmic microwave background radiation for the end of recombination. In order to determine it for the present moment, we can no longer use our approximate solutions of general integro-differential equations. However, considering that after the end of recombination N_e is equal to zero, we can use these equations with the integral term turning to zero. In other words, we obtain common equations for the anisotropy. The solution of system (1) for the polarization will be

$$\beta_0 = \beta_{\rm rec} \cdot \exp\left(-ik\mu(\eta_0 - \eta_{\rm rec})\right).$$

The subscript "rec" indicates that the corresponding value is taken at the moment of recombination.

4. ALLOWANCE FOR ANTENNA EFFECT

The degree of polarization of the cosmic microwave background radiation from one plane wave of scalar disturbances is

$$p_s^2 = \beta_s^2(1-\mu^2)^2,$$

and from one gravitational plane wave is

$$p_{GW}^2 = \beta_{GW}^2((1+\mu^2)^2 + 4\mu^2).$$

Suppose that the receiving antenna pattern is described by the Gauss function

$$d\mathcal{A}(\theta) = \frac{1}{\theta_a^2}\exp\left(-\frac{\theta^2}{2\theta_a^2}\right)\theta d\theta.$$

In this case the full spread of directivity pattern at half-power points (FWHM) is 2.355 θ_a.

We can now write the degree of the cosmic microwave background radiation polarization with allowance for the antenna pattern for the case of stochastic waves in form [26]:

(a) for scalar disturbances

$$p_s^2 = \int d^3k \cdot \frac{1}{2}\int_{-1}^{1} d\mu\beta_s^2((1-\mu^2)^2\, e^{-(kR_h\theta_a)^2(1-\mu^2)},$$

(b) for gravitational waves

$$p_{GW}^2 = \int d^3k \cdot \frac{1}{2}\int_{-1}^{1} d\mu\beta_{GW}^2((1+\mu^2)^2 + 4\mu^2)\, e^{-(kR_h\theta_a)^2(1-\mu^2)}.$$

Here R_h is the contemporary size of the horizon

$$R_h = \frac{c}{H_0}\psi(\Omega, 0).$$

The obtained expression also gives the correct values for $\langle p_s^2\rangle$ and $\langle p_{GW}^2\rangle$ in the case when the spread of the antenna pattern tends to zero ($\theta_a \to 0$):

(a) for scalar disturbances

$$p_s^2 = \frac{8}{15}\int d^3k \cdot \beta_s^2,$$

(b) for gravitational waves

$$p_{GW}^2 = \frac{16}{5}\int d^3k \cdot \beta_{GW}^2.$$

Now it is easy to derive the expression for the root-mean-square polarization value

$$p^2_{r.m.s} = A^2_{SP} \cdot R^2 \int\limits_0^{10} dx x^{n+3} f_s(x, \theta_a) + A^2_{GW} \cdot R^2 \int\limits_0^{10} dx \cdot J^2_{5/2}(x) \cdot x^{n-3} f_{GW}(x, \theta_a).$$

Here the functions $f_s(x, \theta_a)$ and $f_{GW}(x, \theta_a)$:

(a) for scalar disturbances

$$f_s(x, \sigma) = \frac{1}{2} \int\limits_{-1}^{1} d\mu (1-\mu^2)^2 \exp\left(-x^2(1+z_r)\sigma^2(1-\mu^2)\right),$$

(b) for gravitational waves

$$f_{GW}(x, \sigma) = \frac{1}{2} \int\limits_{-1}^{1} d\mu [(1+\mu^2)^2 + 4\mu^2] \exp\left(-x^2(1+z_r)\sigma^2(1-\mu^2)\right)$$

describe the effect of the receiving antenna on the observed degree of the cosmic microwave background radiation polarization;

$$R = \frac{1}{\sqrt{2\pi}} \int\limits_0^{\infty} d\eta \left[\exp(-\tau) - \exp\left(-\frac{3}{10}\tau\right)\right] \left(\frac{\Delta\eta}{\eta_r}\right)$$

is the quantity which depends on the recombination dynamics; $\delta\eta$, η_r denote the recombination interval and the moment of recombination, respectively. We have integrated up to $k\eta_r = 10$ in accordance with our approximation.

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