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OPTICS AND SPECTROSCOPY

COLLINEAR DIFFRACTION OF A LIGHT BEAM ON CONSECUTIVE ACOUSTIC TRAINS

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A system of equations for collinear diffraction of light on two consecutive acoustic trains of finite length with a Gaussian amplitude distribution is derived. Solving these equations yields the amplitude distribution patterns for transmitted and diffracted light under strong acousto-optical interaction. This allows us to calculate the shapes of diffracted light pulses for different train lengths, phase differences, and intertrain distances.

The use of an acoustic train for spectral analysis of optical radiation makes it possible to produce better results than those with a continuous sound signal, but such analysis is restricted by the time of acoustic train passage through a crystal. Using collinear acousto-optical interaction [1–3] with consecutive acoustic trains, it is possible, under certain conditions, to implement continuous spectral analysis of optical radiation.

Before starting to solve such a problem, one has first to investigate light diffraction on two consecutive acoustic trains during their passage through a crystal. In collinear interaction, diffraction occurs all the time a train is propagating inside the acousto-optical cell. If the intertrain distance does not exceed the crystal length L, diffracted light is emitted continuously. There is a time interval in which acousto-optical diffraction occurs simultaneously on two trains. This significantly affects the diffracted light pulse shape at the cell output.

Several publications were devoted to light diffraction on acoustic trains [4, 5], but in all cases their authors considered only the transverse acousto-optical interaction. We are unaware of any theoretical studies of collinear light diffraction on short acoustic trains.

Two identical consecutive acoustic trains of Gaussian shape, of length $2l$, and spaced S apart propagate without energy loss along the x direction at a velocity v relative to the medium. The trains can be represented as $a(x,y,z,t) = a_0 Z(x,y,z,t)$, where

$$Z(x,y,z,t) = G(x,y,z)\left(\exp\left[-\frac{(vt-x)^2}{l^2}\right]\exp\{j(\Omega_0 t - K_0 x)\} + \exp\left[-\frac{(vt-x-S)^2}{l^2}\right]\exp\{j(\Omega_0 t - K_0[x-S] + \varphi_0)\}\right) + \text{ c.c.}, \tag{1}$$

where

$$G(x,y,z) = \frac{1}{\sqrt{1-jD_1x}}\frac{1}{\sqrt{1-jD_2x}}\exp\left[-\frac{y^2}{R_1^2(1-jD_1x)} - \frac{z^2}{R_2^2(1-jD_2x)}\right];$$

a_0 is the wave amplitude at the cell input (at $x=0$); K_0 and Ω_0 are the acoustic train's central wave number and central frequency, respectively; R_i are its transverse dimensions at $x=0$ and $t=0$; $D_i = 2W_i/K_0R_i$ are the acoustic train's divergences in the y and z directions; W_i are the characteristics of anisotropic transverse spreading; φ_0 is the initial phase of the second train; and $i = 1, 2$.

The propagation of acoustic train (1) is accompanied by a wave of elastic deformations described by the medium deformation tensor $S_{lm}Z(x,y,z,t)$. The space and time propagation of these deformations conforms to (1), and their magnitude is proportional to a_0. The deformation wave changes the medium's refractivities due to the optoelasic effect [6, 7] described by the tensor p_{jklm}. The variation of the medium's permittivity tensor components under the acoustic field effect has the form $\Delta\varepsilon_{jk} = -N_j^2 N_k^2 \sum_{l,m=1}^{3} p_{jklm}S_{lm}$. Here N_j and N_k are the primary refractivities of the medium; j, k, l, and m are the coordinate subscripts.

Light diffraction on sound in this case is described by a wave equation of the form

$$\operatorname{curl}\operatorname{curl}\mathbf{E} + \frac{\widehat{\varepsilon}_0}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = -\frac{\Delta\widehat{\varepsilon}}{c^2}\frac{\partial^2 Z\mathbf{E}}{\partial t^2}, \tag{2}$$

where $\mathbf{E}$ is the light wave electric vector, $\widetilde{\varepsilon}_0$ is the medium permittivity tensor in the absence of sound, and $\Delta\widetilde{\varepsilon}$ is the sound-induced variation of $\widetilde{\varepsilon}_0$. It should be noted that if the light beam has finite dimensions, then $\operatorname{curl}\operatorname{curl}\mathbf{E} \neq \nabla^2\mathbf{E}$, because $\operatorname{grad}\operatorname{div}\mathbf{E}$ cannot be assumed to be zero even in an isotropic medium.

It is well known that in collinear diffraction, polarization of diffracted light beam is normal to that of incident light. Therefore, in the region of light–sound interaction it is natural to represent the light beam as a sum of transmitted and diffracted beams with orthogonal polarizations,

$$\mathbf{E} = E_t(x,y,z)\mathbf{e}_t\exp[j(k_t x - \omega_t t)] + E_d(x,y,z)\mathbf{e}_d\exp[j(k_d x - \omega_d t)], \tag{3}$$

where $\mathbf{e}_t$, $\mathbf{e}_d$, ω_t, ω_d, n_t, n_d, $k_d = \omega_d n_d/c$ and $k_t = \omega_t n_t/c$ are the polarization directions, frequencies, refractivities, and mean wave numbers of transmitted and diffracted light, respectively.

Let us substitute $\mathbf{E}$ of (3) into (2), use the diffraction condition $\omega_d = \omega_t + \Omega_0$, and equate the amplitudes of $\exp\{j\omega_t t\}$ and $\exp\{j\omega_d t\}$. Neglecting the quantities $\partial^2 E_t/\partial t^2$ and $\partial^2 E_d/\partial x^2$, we apply the two-dimensional Fourier transform to both parts of (2) in the yz-plane. Omitting mathematical manipulations similar to those described in [8, 9] and taking the orthogonality of polarizations $\mathbf{e}_t$ and $\mathbf{e}_d$ into account, we obtain a system of scalar equations for the spectra of transmitted and diffracted light, U_t and U_d,

$$2jk_d\frac{\partial U_d}{\partial x} + k_y^2 U_d = q_1\exp\{-j\eta x\}\iint_{-\infty}^{\infty} A(K_y,K_z,x,y)U_t(k_y+K_y,k_z+K_z,x)\,dK_y dK_z, \tag{4}$$

$$2jk_t\frac{\partial U_t}{\partial x} + k_z^2 U_t = q_2\exp\{j\eta x\}\iint_{-\infty}^{\infty} A^*(K_y,K_z,x,t)U_d(k_y-K_y,k_z-K_z,x)\,dK_y dK_z, \tag{5}$$

where $q_1 = k_t^2(\mathbf{e}_t\Delta\widehat{\varepsilon}\mathbf{e}_d)/n_t^2$; $q_2 = k_d^2(\mathbf{e}_d\Delta\widehat{\varepsilon}\mathbf{e}_t)/n_d^2$; $\eta = k_t + K_0 - k_d$; k_y, k_z, K_y, and K_z are the transverse components of the light and sound wave vectors, respectively; and $A(K_y,K_z,x,t)$ is the Fourier spectrum of the sound train. The interrelation between U_t and E_t, as well as between U_d and E_d, has the form

$$U(k_y,k_z,x) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} E(x,y,z)\exp[-j(k_y y + k_z z)]\,dy\,dz, \tag{6}$$

and that between A and Z has the form

$$A(K_y,K_z,x,t) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} Z(x,y,z,t)\exp[-j(K_y y + K_z z)]\,dy\,dz. \tag{7}$$

Let us represent the functions U_t and U_d by

$$U_t(k_y,k_z,x) = f_t(x)\exp\{jxk_z^2/2k_t\}\exp\{-(k_y^2+k_z^2)r_t^2/4\}, \tag{8}$$

$$U_d(k_y,k_z,x) = f_d(x)\exp\{-jxk_y^2/2k_d\}\exp\{-(k_y^2+k_z^2)r_d^2/4\}. \tag{9}$$

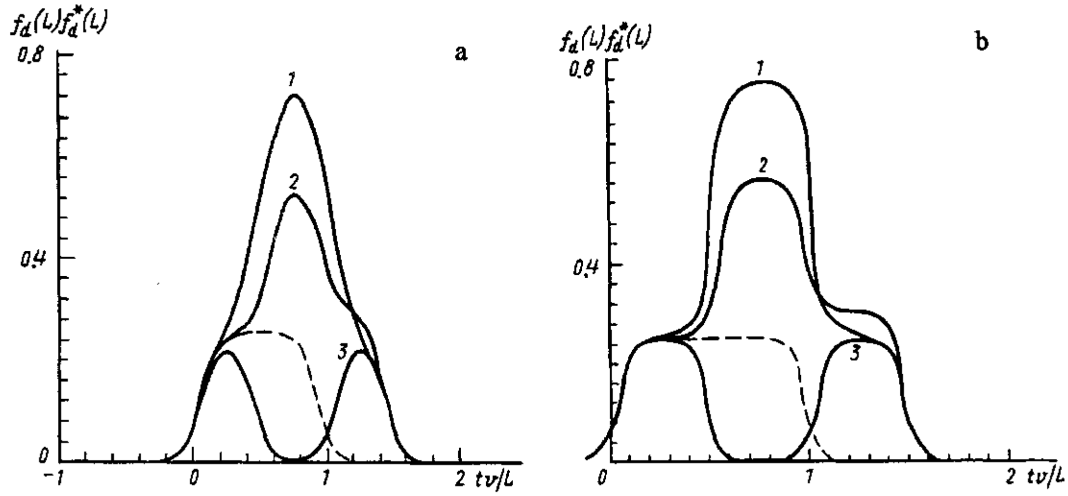

Fig. 1

Diffracted light pulse amplitude $f_d(L)f_d^*(L)$ as a function of dimensionless time tv/L for varying phase differences between trains of length $l/L = 0.2$ (a) and 0.1 (b), spaced half the crystal length ($S/L = 0.5$) apart. $DL = 1$, $\varphi = 0$ (*1*), $\pi/2$ (*2*), and π (*3*).

Here $f_t(x)$ and $f_d(x)$ are the light beam amplitudes dependent on x and taken along the axis at $k_y = k_z = 0$. The radii of the incident r_t and diffracted r_d Gaussian light beams in (8), (9) relate as $r_d = r_t/\sqrt{(1 + r_t^2/R^2)}$.

Substituting (8), (9), and (1) into (4) and (5) and assuming $R = R_1 = R_2$ and $D = D_1 = D_2$, we can analytically calculate the integrals in dK_y and dK_z in the right-hand sides. Neglecting the light beam radii variations, we obtain a system of two first-order differential equations that describe collinear light diffraction on Gaussian acoustic trains in the case of strong acousto-optical interaction,

$$\frac{df_d}{dx} = -j\frac{q_1 f_t(x)\exp\{-jx\eta\}}{(1-jDx)+r_t^2/R^2}\left(\exp\left[-\frac{(vt-x)^2}{l^2}\right]+\exp\left[-\frac{(vt-x-S)^2}{l^2}\right]\exp\{j\varphi(S)\}\right), \quad (10)$$

$$\frac{df_t}{dx} = -j\frac{q_2 f_d(x)\exp\{-jx\eta\}}{(1+jDx)+r_d^2/R^2}\left(\exp\left[-\frac{(vt-x)^2}{l^2}\right]+\exp\left[-\frac{(vt-x-S)^2}{l^2}\right]\exp\{j\varphi(S)\}\right), \quad (11)$$

where $\varphi(S) = K_0 S + \varphi_0$ is the intertrain phase difference. The interaction domain in (10), (11) is $0 < x < L$, therefore henceforth we will use the dimensionless quantities x/L and $qL = q_1 L = q_2 L$. All calculations were carried out at zero detuning ($\eta = 0$), the beam radii ratio $r_t/R = 0.1$, and $D_1 L = D_2 L = DL$.

The diffracted light pulse shape strongly depends on the intertrain phase difference. Figure 1a shows the light beam shapes for different train phase differences. The intertrain distance is half the crystal length, the trains are five times shorter than the crystal length. The dashed curve shows the light beam shape in the case of a single train. For zero phase difference (curve *1*) the amplitude of diffracted light is more than twice higher than in the case of a single train. The light pulse duration is also longer because it is determined by the train width and the intertrain distance. Increasing the phase difference to $\pi/2$ (curve *2*) makes diffraction less efficient and distorts the curve shapes, though does not affect diffracted pulse length. At the phase difference of π (curve *3*), diffraction on one train is opposite in phase to that on the other. The light pulse splits into two pulses corresponding to those time instants at which only one train is traveling in the crystal. The same situation is depicted in Fig. 1b for a twice shorter acoustic train. All results are quite similar, but the flat tops of all diffracted pulses are more pronounced.

These data demonstrate that the acoustic train phase difference plays an important role in diffraction efficiency. Figure 2 shows the maximum amplitude $f_d(L)$ proportional to the diffraction efficiency as a function of the intertrain phase difference. Let us consider the case of the trains located symmetrically with respect to the crystal center. As seen in Fig. 2, the best diffraction efficiency is achieved if the phase difference magnitude does not exceed 0.4π. The efficiency falls abruptly at $|\varphi| = \pi/2$ decreasing by more than three times as π approaches $|\varphi|$.

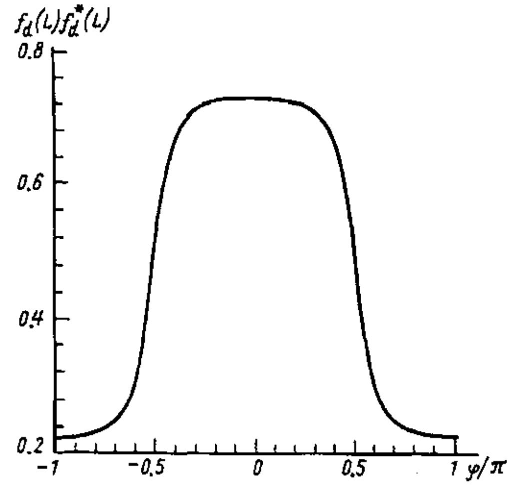

Fig. 2
Maximum amplitude of diffracted light pulse $f_d(L)f_d^*(L)$ as a function of intertrain phase difference φ/π: $l/L = 0.1$; $DL = 1$.

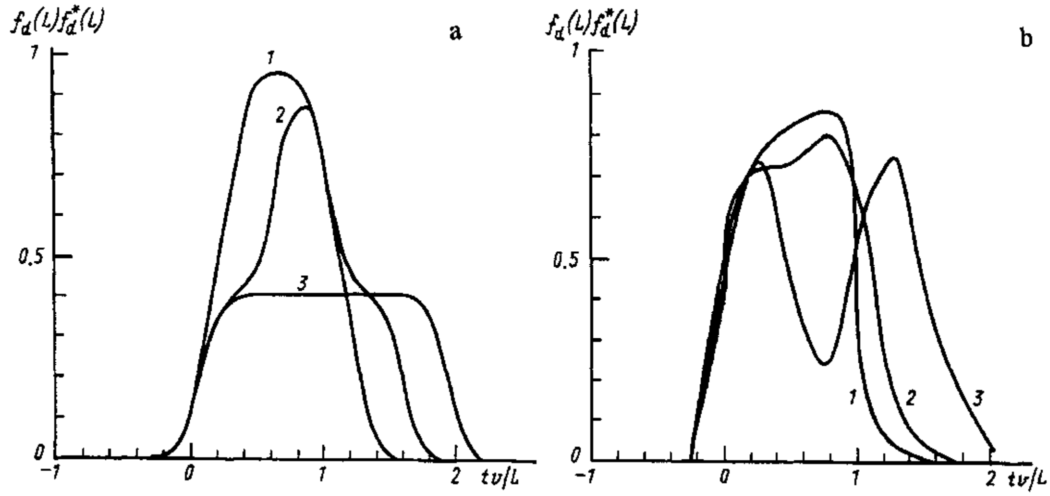

Fig. 3
Diffracted light pulse amplitude $f_d(L)f_d^*(L)$ as a function of dimensionless time tv/L for varying distance between trains of length $l/L = 0.2$ with divergences $DL = 1$ (*a*) and 5 (*b*); $S/L = 0.25$ (*1*), 0.5 (*2*), and 1 (*3*).

Based on these calculations, our subsequent investigations were carried out at the optimum phase difference $\varphi = 0$. Figure 3*a* shows the diffracted light pulse as a function of the intertrain distance for the case of each train width much shorter than the crystal length. Variation of the intertrain distance changes the ratio of the time intervals during which one or two trains simultaneously exist in the crystal. Curve *1* corresponds to two trains following immediately one after the other, making the diffraction efficiency close to its maximum value and the light pulse duration close to its minimum. At $S = L/2$ (curve *2*), there appears an interval of time tv/L during which diffraction occurs on a single train, therefore the curve assumes a typical double-step shape. The case of the intertrain distance equal to the crystal length (curve *3*) corresponds to diffraction on two consecutive single trains.

For increased train divergence ($DL = 5$) and the corresponding tenfold increase of the input acoustic intensity (Fig. 3*b*), the shapes of diffracted light pulses significantly change as compared to respective curves

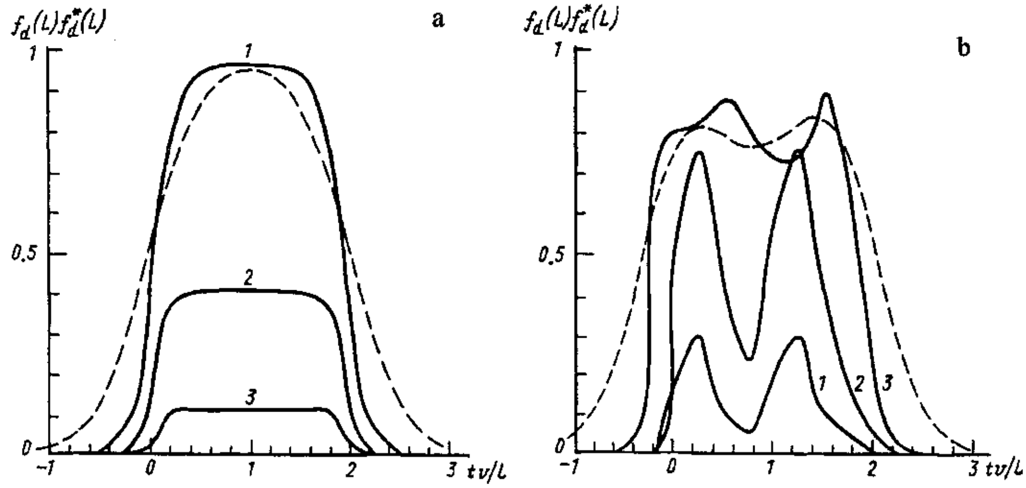

Fig. 4

Diffracted light pulse amplitude $f_d(L)f_d^*(L)$ as a function of dimensionless time tv/L for varying train length for intertrain distances equal to the crystal length ($S/L = 1$), divergences $DL = 1$ (*a*) and 5 (*b*); $l/L = 0.4$ (*1*), 0.2 (*2*), and 0.1 (*3*).

in Fig. 3*a*, but the total durations of these pulses remain the same. If the intertrain distance exceeds but slightly the train length, the diffracted pulse remains single (curves *1* and *2*), otherwise it splits into two consecutive pulses (curve *3*).

Figure 4*a* shows the shapes of diffracted light pulses for different acoustic train lengths with intertrain distances equal to the crystal length. Solid curves *1–3* in Fig. 4*a* correspond to train durations shorter than half the crystal length, which means that consecutive trains do not overlap. As seen in the figure, in this case, the diffracted light pulses have the same shape independent of the train length. All curves feature flat pulse tops, because most of the time diffraction occurs on one of the two trains. However, the diffracted light pulse intensity sharply increases with growing train duration. It is obvious that with increasing number of consecutive trains spaced L apart, the diffracted pulses (curves *1–3*) will turn into continuous light of corresponding amplitude. If the trains overlap (their durations exceed half the crystal length), the flat top disappears in the diffracted pulse. This is graphically illustrated by the dashed curve in Fig. 4*a*, calculated for the case of train width equal to the crystal length, $l = L = S$. In this case, the maximum pumping is achieved with a four times smaller input acoustic intensity.

If the train divergence increases to $DL = 5$ (see Fig. 4*b*), the flat pulse top is distorted because the sound amplitude on the axis decreases as the train propagates. The diffracted pulse remains single for $l > 0.4L$, splitting into two pulses for smaller values of l. Thus, for a sequence of acoustic trains with high divergences following one another at a distance equal to the crystal length, the diffracted pulses cannot be transformed into continuous light.

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