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SOLID-STATE PHYSICS

SOLUTION OF ONE INVERSE PROBLEM OF PHOTOTHERMAL DIAGNOSTICS WITH THE USE OF THE TYKHONOFF REGULARIZATION METHOD

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The problem of nondestructive photothermal diagnostics of one-dimensional medium is considered. An algorithm based on numerical minimization of the objective function and spline approximation of the medium parameters is developed. The algorithm reproduces satisfactorily the shape of nonuniformities.

Photothermal diagnostics is an effective technique of nondestructive testing used to reveal nonuniformities in thin near-surface layers of samples. The essence of the method is that the sample surface is heated by intensity-modulated laser radiation, and the surface temperature or, more exactly, its component oscillating at the modulation frequency, is recorded. Although the whole sample is heated during the measurements, the oscillating temperature component (heat wave) does not penetrate deep, and its value on the surface contains information on a thin near-surface layer of the substance only. By solving the corresponding inverse problem, we can diagnose nonuniformities present in this layer.

Various approaches to solving the inverse problem have been extensively studied during the past decade yet under certain restrictive assumptions. For instance, in [1], the first Born approximation was used; in [2, 3], explicit constraints were imposed on the heat capacity and heat conductivity derivatives; in [4], the medium was represented by a finite number of uniform layers; and in [5], one of the heat capacity and heat conductivity profiles was constant, while the other consisted of linearly nonuniform layers. In [6], no explicit constraints were imposed on nonuniformities, but the algorithm reproduced nonuniformities from the obtained data at a single fixed modulation frequency.

Therefore, it appears desirable to develop an approach free of stringent constraints on nonuniformity smoothness and smallness. In [7, 8], we suggested such an approach applicable to one-dimensional nonuniformities. It was based on analytically relating the heat wave on the surface to the heat conductivity and heat capacity profiles with the use of discretization on a mesh. In other words, the surface response was obtained as an explicit function of medium parameters. Then we applied the approach usually employed in such problems, i.e., minimization of the objective function, or of the deviation of the response calculated at the given medium parameters from the experimental data. In this technique, the objective function allows for analytical calculation of its gradient, thus a very effective method of conjugate gradients was applied.

Even with all its advantages, the suggested approach had one important shortcoming: it could hardly be extended to the case of multi-dimensional nonuniformities. In this work, the above problem is solved somewhat differently with allowance for such an extension. Our approach is based on spline approximation of the medium thermophysical characteristics and the objective function minimization by the direct search method involving regularization. In this case, no analytical calculation of the objective function gradient is required.

1. PROBLEM STATEMENT

We consider the inverse problem in the following geometry. The nonuniform medium occupies the half-space $x > 0$, and the thermophysical parameters depend on the coordinate x only. The medium surface $x = 0$ is irradiated by harmonically modulated laser radiation with a cyclic frequency ω. The radiation intensity is

$$I(t) = I_0 + \mathrm{Re}(I\, e^{i\omega t}),$$

where I_0 and I are the mean intensity and the modulation amplitude, respectively ($I_0 > I$). Heating is performed at the point $r = 0$, and the temperature is measured at the point r ($r^2 = y^2 + z^2$).

We define the complex amplitude of the heat wave $U(x, r, \omega)$ by

$$U(x, r, t) = U_0 + \mathrm{Re}[U(x, r, \omega)\, e^{i\omega t}]$$

and write the following equation for the amplitude:

$$k(x)\frac{1}{r}U_r + k(x)U_{rr} + k'(x)U_x + k(x)U_{xx} = i\omega c(x)U \tag{1}$$

with the boundary conditions

$$U_x\Big|_{x=0} = -\frac{I}{k(0)}\frac{\delta(r)}{2\pi r}, \quad U_x\Big|_{x\to\infty} = 0, \tag{2}$$

where $k(x)$ is the heat conductivity and $c(x)$ is the unit volume heat capacity. Let us introduce the $T(x, s, \omega)$ complex function related to the initial complex amplitude by a zeroth-order Hankel transformation,

$$T(x, s, \omega) = \int_0^\infty J_0(sr)U(x, r, \omega)r\,dr. \tag{3}$$

The equation for the $T(x, s, \omega)$ spectral amplitude is an equation in total derivatives,

$$T_{xx} + \frac{k'(x)}{k(x)}T_x = \left(i\omega\frac{c(x)}{k(x)} + s^2\right)T, \tag{4}$$

and the boundary conditions are

$$T_x\Big|_{x=0} = -\frac{1}{2\pi k(0)}, \quad T_x\Big|_{x\to\infty} = 0. \tag{5}$$

The inverse problem is as follows. Given the $T(0, s, \omega)$ value easily found from an experimental surface temperature profile with the use of Hankel transformation (3). Calculate the values of $c(x)$ and $k(x)$.

2. PROBLEM SOLVING ALGORITHM

The solution breaks down into three stages, *viz.*, (i) discretization of variables and functions describing the medium; (ii) selection of an algorithm for calculating surface response in terms of the known medium parameters; and (iii) selection of an objective function and its numerical minimization.

Discretization. Equation (4) contains functions $c(x)/k(x) = a(x)$ and $k'(x)/k(x) = b(x)$. Let the minimization variables be $a(x_q)$ and $b(x_q)$, where x_q ($q = 1, \ldots, N_q$) are the x values on a spatial mesh. The choice of the number N_q of points is determined by a compromise between the requirements that, first, the mesh width be fairly small (smaller than the nonuniformity characteristic size), and, second, the number N_q of mesh nodes be as small as possible to reduce computational expenditures. In practice, we used N_q of about 20.

Algorithm for solving the direct problem. The response value was calculated by the method of continued fractions developed by us in [7, 8]. The required accuracy was provided by using a step substantially smaller than the principal mesh width x_q. The $a(x_j)$ and $b(x_j)$ values at subsidiary mesh nodes x_j ($j = 1, \ldots, N_j$) were calculated from $a(x_q)$ and $b(x_q)$ with the use of the bicubic spline approximation.

Typically, the number of subsidiary nodes $N_j = 250$. Note that the direct mesh problem can also be solved by standard methods (e. g., the sweep method) because no analytical calculation of the objective function gradient is required.

Objective function minimization. Consider the normalized deviation of the calculated response from the required value (the residual function) as a function of medium parameters,

$$F(\{a_q\}, \{b_q\}) = \sum_{l,p} \left| \frac{T(0, s_p, \omega_l) - T^{\mathrm{exp}}(0, s_p, \omega_l)}{T^{\mathrm{hom}}(0, s_p, \omega_l)} \right|^2 . \tag{6}$$

Here, $T^{\mathrm{exp}}(0, s, \omega)$ is the experimental response; $T^{\mathrm{hom}}(0, s, \omega)$ is the solution for a homogeneous medium; the subscripts p and l refer to the meshes based on parameter s and frequency ω, respectively. If we set the nonzero measurement error δ, then any medium (i. e., the set $\{a_q\}$ and $\{b_q\}$) such that its response deviates from the required value by less than δ can be considered a solution. We cannot guarantee that all these solutions be close to the true nonuniformity profile. Indeed, our computations show that at residual values (6) of about 1%, profiles with gigantic oscillations radically different from the true profile are obtained, that is, the problem is ill-posed. For this reason, we should apply the Tykhonoff approach and modify the objective function according to [9] as follows:

$$T(\{a_q\}, \{b_q\}) = F(\{a_q\}, \{b_q\}) + \alpha \sum_q \left((a_q - a_0)^2 + b_q^2\right) . \tag{7}$$

The second term is the square of the distance between the current and the uniform profile multiplied by the regularization parameter α; a_0 corresponds to a uniform medium. At a given $\delta > 0$, the α value should be adjusted to satisfy the equality $F(\{a_q^{\mathrm{min}}\}, \{b_q^{\mathrm{min}}\}) = \delta^2$, where $\{a_q^{\mathrm{min}}\}, \{b_q^{\mathrm{min}}\}$ is the element at which $T(\{a_q\}, \{b_q\})$ attains a minimum.

The numerical minimization was carried out by the method of principal axes [10], which is one of the most effective direct search algorithms. Within each cycle of iterations, the program performed one-dimensional minimizations along the principal axes directions found at the preceding step, and then constructed a quadratic form using the obtained points. The quadratic form point of minimum was used in the next cycle.

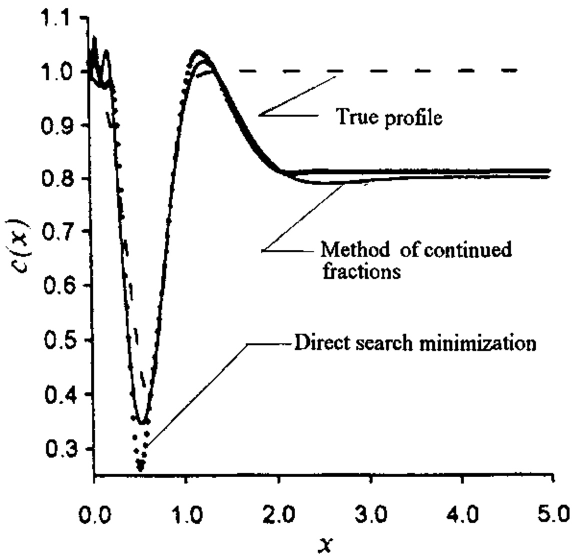

Fig. 1

Reproduction of heat capacity coefficient.

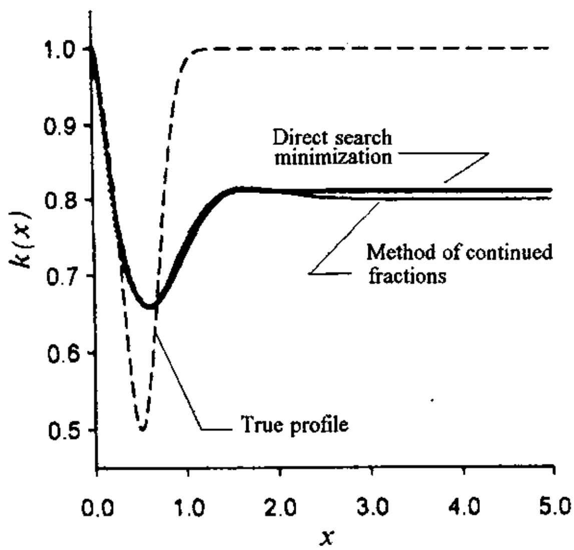

Fig. 2

Reproduction of heat conductivity coefficient.

3. NUMERICAL ANALYSIS RESULTS

The described algorithm was tested on an example of Gaussian nonuniformity. The results of the reconstruction of heat conductivity and heat capacity profiles are given in Figs. 1 and 2. For comparison, the profiles obtained by the method of continued fractions are also shown. First, we note that the reconstruction is possible for $0 < x < 1.5$ only. This region corresponds to the maximum depth of heat wave penetration because experimental data does not contain information on deeper layers. The heat capacity profile $c(x)$ is reproduced much better than the heat conductivity profile $k(x)$. The calculations were performed for an undistorted response.

CONCLUSION

Let us summarize the results briefly. The suggested procedure makes it possible to reconstruct the form of nonuniformity in the near-surface layer of samples. It does not impose stringent constraints on smallness and smoothness of thermophysical parameters. The minimization procedure (the method of principal axes) allows the suggested approach to be extended to multidimensional problems.

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