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NUMERICAL METHOD OF SOLVING THE PROBLEM OF ACOUSTIC-GRAVITY WAVES PROPAGATION IN THE ATMOSPHERE UP TO IONOSPHERIC HEIGHTS

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A numerical algorithm and software have been developed for solving a set of one-dimensional nonlinear equations of geophysical hydrodynamics that describe generation of acoustic–gravity waves resulting from earthquakes and strong explosions and their propagation to a great height in the atmosphere.

INTRODUCTION

The study of generation and propagation of acoustic–gravity waves in the terrestrial atmosphere is a major problem of modern physics of atmosphere. The conventional methods of the atmosphere state observation are improved, new means and measurement procedures are developed, algorithms and software for numerical calculations are developed, new problems of taking into account nonlinear effects in the emission and propagation of acoustic–gravity waves are posed.

It has been found that the main sources of such waves in the atmosphere are the following phenomena: earthquakes, volcanic eruptions, hurricanes, storms, solar eclipses, jet flows, polar and equatorial current systems, meteors, strong explosions, launching of high-power rockets [1, 2]. The efficiency of mechanisms of acoustic–gravity wave generation is evaluated by solving the respective equations of hydrodynamics with sources of mass, energy and momentum involved. In studies of atmospheric disturbances caused by surface Rayleigh waves or ocean surface waves under the effect of earthquakes or underground explosions, these sources can be simulated by preset motions of the interface [1]. It is believed that acoustic–gravity waves are also generated before strong earthquakes [3].

Because of an exponential decrease of the atmospheric density with height, the amplitude of acoustic–gravity waves may increase considerably in the upper atmosphere, where they can affect the ionosphere by varying the distribution of neutral and charged particles [4]. Present-day methods of studying the upper atmosphere, such as radiotomography or incoherent scattering, make it possible to register acoustic–gravity waves in the ionosphere in large spatial regions [5].

The basic goal of this work is to develop a numerical method for solving a set of hydrodynamic equations, which describes propagation in the atmosphere and ionosphere of acoustic–gravity waves generated by earthquakes and strong explosions. As solving this problem numerically involves some specific difficulties, here we investigate in detail the solutions of such a set of equations together with the initial and boundary conditions for the one-dimensional case. This made it possible to choose a stable and effective numerical method for solving similar problems in the two- and three-dimensional cases.

SET OF EQUATIONS

The initial system of analysis of acoustic–gravity waves generation and propagation is the set of Euler's conservation equations

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0, \\ \rho\left(\dfrac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}, \boldsymbol{\nabla})\,\mathbf{v}\right) = -\nabla p + \rho \mathbf{g} + \mathbf{F}^d, \\ \rho\left(\dfrac{\partial (c_v T)}{\partial t} + (\mathbf{v}, \boldsymbol{\nabla})(c_v T)\right) = -p(\boldsymbol{\nabla}, \mathbf{v}) + Q^d, \\ p = \dfrac{\rho}{m_0} RT. \end{cases} \tag{1}$$

The first equation is the continuity equation, the second is the moment-of-momentum equation, the third is the energy equation, and the last one is the ideal gas state equation. The Coriolis force is insignificant for the fast motions under consideration, therefore, it is ignored. Here, ρ is the density, T is the temperature, p is the pressure, $\mathbf{v}$ is the air particle velocity, g is the free fall acceleration, $\mathbf{F}^d$ is the viscous force, Q^d is the heat absorbed due to wave dissipation, c_v is the specific heat of gas at constant volume, m_0 is the relative molecular mass of air, and R is the universal gas constant.

Internal gravity waves are the most intense part of the spectrum of acoustic–gravity waves. The following dissipative mechanisms are essential for these waves below the turbopause: molecular viscosity, thermal conductivity and interaction with the background turbulence of the atmosphere [6]. The studies show that here molecular viscosity and thermal conductivity cannot produce a strong effect on the momentum and energy of gravity waves, because damping of such large-amplitude wave motions requires time of the order of several days. But we are interested in processes occurring within a few hours, therefore, in our model, we shall consider only the effect of turbulent viscosity up to the turbopause height. Above that, the major mechanism is the ionic deceleration [6, 7]. Let us combine the two different forces (viscosity and ionic deceleration) into the force $\mathbf{F}^d$, which we take in the Rayleigh form [6]: $F^d = -\alpha \mathbf{v}$. Here, α is the resistance coefficient, whose value corresponds to different forces at different heights. For the sake of simplicity, we use the mean value of α, which varies with height so that the resistance coefficient-to-density ratio remains constant: $\alpha/\rho = \text{const}$. The main dissipative mechanism affecting wave energy is thermal conductivity [6]. Thus, $Q^d = K\nabla^2 T$ in set of equations (1); K is the coefficient of thermal conductivity of the air.

We now decompose each thermodynamic parameter into two parts—a background stationary part denoted by the subscript 0 and a disturbed part designated by a prime

$$\rho = \rho_0 + \rho', \quad T = T_0 + T', \quad p = p_0 + p'. \tag{2}$$

For a numerical solution, equation (1) is conveniently reduced to dimensionless form. To do this, the physical quantities are normalized as follows:

$$\rho \to \frac{\rho}{\rho_s}, \quad p \to \frac{p}{p_s}, \quad T \to \frac{T}{T_s}, \quad \mathbf{v} \to \frac{\mathbf{v}}{c_s}, \quad \mathbf{r} \to \frac{\mathbf{r}}{H}, \quad t \to t\frac{c_s}{H}, \quad m_0 \to \frac{m_0}{m_s}. \tag{3}$$

The subscript s indicates the value of the respective parameter at the Earth's surface, H is the height of a homogeneous atmosphere (8400 m), and c_s is the sound velocity at the Earth's surface (340 m/s). On rearranging (2) and (3) for the one-dimensional case, we obtain a set of hyperbolic equations and the ideal gas state equation

$$\begin{cases} \dfrac{\partial \rho'}{\partial t} = -\dfrac{\partial}{\partial z}\left[(\rho_0 + \rho')\, w\right], \\ \dfrac{\partial w}{\partial t} = -w\dfrac{\partial w}{\partial z} - A_1 \dfrac{1}{\rho_0 + \rho'}\dfrac{\partial p'}{\partial z} - A_2 \dfrac{\rho'}{\rho_0 + \rho'} - A_3 w, \\ \dfrac{\partial T'}{\partial t} = -w\dfrac{\partial (T_0 + T')}{\partial z} - A_4 \dfrac{\partial w}{\partial z} + A_5 \dfrac{\partial^2 (T_0 + T')}{\partial z^2}, \\ p' = (\rho_0 T' + \rho' T_0 + \rho' T')/m_0, \end{cases} \tag{4}$$

where the z-axis is directed vertically upward, w is the velocity vertical component of the atmosphere particle motion, and $A_1 = \frac{p_s}{\rho_s c_s^2}$, $A_2 = \frac{H g_s}{c_s^2}$, $A_3 = \frac{H\alpha}{\rho_s c_s^2}$, and $A_4 = \frac{R}{m_s c_v}$, $A_5 = \frac{K}{c_s c_v \rho_s H}$ are the dimensionless constants.

The Gaussian function can be used to simulate the displacement of the Earth's solid surface caused by shallow earthquakes and explosions. The time derivative of this function gives the velocity value

$$w(0,t) = -\frac{2D_0}{\Delta t}\left(\frac{t-\bar{t}}{\Delta t}\right)\exp\left(-\left(\frac{t-\bar{t}}{\Delta t}\right)^2\right). \tag{5}$$

The values of the parameters D_0 and Δt that characterize the amplitude and frequency of disturbance are discussed in detail in [8]. The velocity of motion of air particles at the Earth's surface is equal to that of the Earth's surface. This boundary condition provides the transfer of a disturbance from the solid medium to the atmosphere.

NUMERICAL METHOD

Methods employed for modeling equations can be divided into spectral methods that use the expansion of unknown functions in spherical harmonics, and difference methods. In a number of cases, spectral methods are more suitable for analysis and interpretation of results, however, their application requires prior linearization of the equations (or cumbersome nonlinearity iteration procedure), which makes difference methods more preferable in such problems.

For a numerical solution of set of equations (4), it is required to select an effective method of approximation suitable for a particular problem. Since the disturbances are very fast ($\Delta t = 0.05$–0.25 s), and the time of wave propagation to high altitudes is more than one hour, one has to integrate approximated equations with a small time step which requires tens of thousands of iterations. In view of this circumstance and nonlinearity of the set of equations, we came to a conclusion that the explicit difference method is most suitable for integrating hydrodynamic equations. When solving set (4), the following considerations should be taken into account.

1. As the method is explicit, the Courant–Friedrichs–Levi condition, i. e., $c\frac{\Delta t}{\Delta z} < 1$, should be satisfied to ensure stability of set (4) [9], where c is the sound velocity, and Δt and Δz are, respectively, the grid time and space steps.

2. Equations (4) are solved with respect to density and temperature variations that may be either positive or negative, therefore, there is no need to use the numerical method which makes the solution positive [10].

3. The nonlinear term in the velocity equation must be approximated by a special method to ensure stability [9].

4. In the third equation of set (4) the advection term must be approximated in conservative form [9].

5. It is known that the background density ρ_0 of the atmosphere undergoes very fast variations with height and that high gradients may cause non-physical oscillations if the method of the second or higher orders is used. This circumstance should be remembered in selecting a difference method [11].

In view of the above circumstances, on applying different methods, we selected an algorithm of transfer calculation with current correction (FCT method) [10]. Within 10–15 recent years, this method has been applied with much success for various problems of hydrodynamics. We shall demonstrate the application of this method for the first equation of the set. Then

$$\overline{\rho'^n_k} = \rho'^n_k - \frac{\Delta t}{4\Delta z}\left[\left(\rho'^n_k + \rho'^n_{k+1}\right)\left(w^n_k + w^n_{k+1}\right) - \left(\rho'^n_k + \rho'^n_{k-1}\right)\left(w^n_k + w^n_{k-1}\right)\right],$$

where $k = 0, 1, \ldots, K$ is the number of the grid mesh along the vertical axis, and $n = 0, 1, \ldots, N$ is the number of the grid mesh in terms of time. $\overline{\rho'^n_k}$ denotes the intermediate value of density disturbance. This equation should involve the diffusion part, which provides stability

$$\tilde{\rho}'^n_k = \overline{\rho'^n_k} + \nu\left(\rho'^n_{k+1} - 2\rho'^n_k + \rho'^n_{k-1}\right),$$

where ν is the dimensionless diffusion coefficient. At the second stage, large numerical diffusion is eliminated, but so that to prevent the appearance of non-physical extrema in the solution. A special constraint imposed on the anti-diffusion currents will serve the purpose. They are calculated as follows

$$f^c_{k+1/2} = S_{k+1/2}\max\left\{0, \min\left[\left|f^{ad}_{k+1/2}\right|, S_{k+1/2}\left(\tilde{\rho}'^{n}_{k+2} - \tilde{\rho}'^{n}_{k+1}\right), S_{k+1/2}\left(\tilde{\rho}'^{n}_{k} - \tilde{\rho}'^{n}_{k-1}\right)\right]\right\},$$

where $f^{ad}_{k+1/2} = \mu\left(\overline{\rho'^{n}_{k+1}} - \overline{\rho'^{n}_{k}}\right)$ is the antidiffusion flow, $S_{k+1/2} = \mathrm{sign}\left(f^{ad}_{k+1/2}\right)$, the two other expressions are the corrections, and μ is the dimensionless antidiffusion coefficient. When calculating antidiffusion flows through the end boundaries, the corrections corresponding to finite differences beyond the simulation domain drop out of the calculation. Let us perform the anti-diffusion stage

$$\rho'^{n+1}_{k} = \tilde{\rho}'^{n}_{k} - f^c_{k+1/2} + f^c_{k-1/2}.$$

The final value of density disturbance at the time layer $n+1$ is ρ'^{n+1}_k. The velocity and temperature disturbance are found in the same way. The nonlinear term in the moment-of-momentum equation is approximated as follows [9]

$$w\frac{\partial w}{\partial z} = \frac{1}{3}w^n_k\left(\frac{w^n_{k+1} - w^n_{k-1}}{2\Delta z}\right) + \frac{1}{3}\left(\frac{\left(w^n_{k+1}\right)^2 - \left(w^n_{k-1}\right)^2}{2\Delta z}\right).$$

The advection term in the energy equation will be [9]

$$w\frac{\partial T'}{\partial z} = \frac{1}{4}\left[\left(w^n_{k+1} + w^n_k\right)\left(\frac{T'^n_{k+1} - T'^n_k}{\Delta z}\right) + \left(w^n_k + w^n_{k-1}\right)\left(\frac{T'^n_k - T'^n_{k-1}}{\Delta z}\right)\right].$$

The terms related to medium sources and thermal conductivity are approximated in the conventional central-difference form. Pressure disturbances are found from the ideal gas state equation.

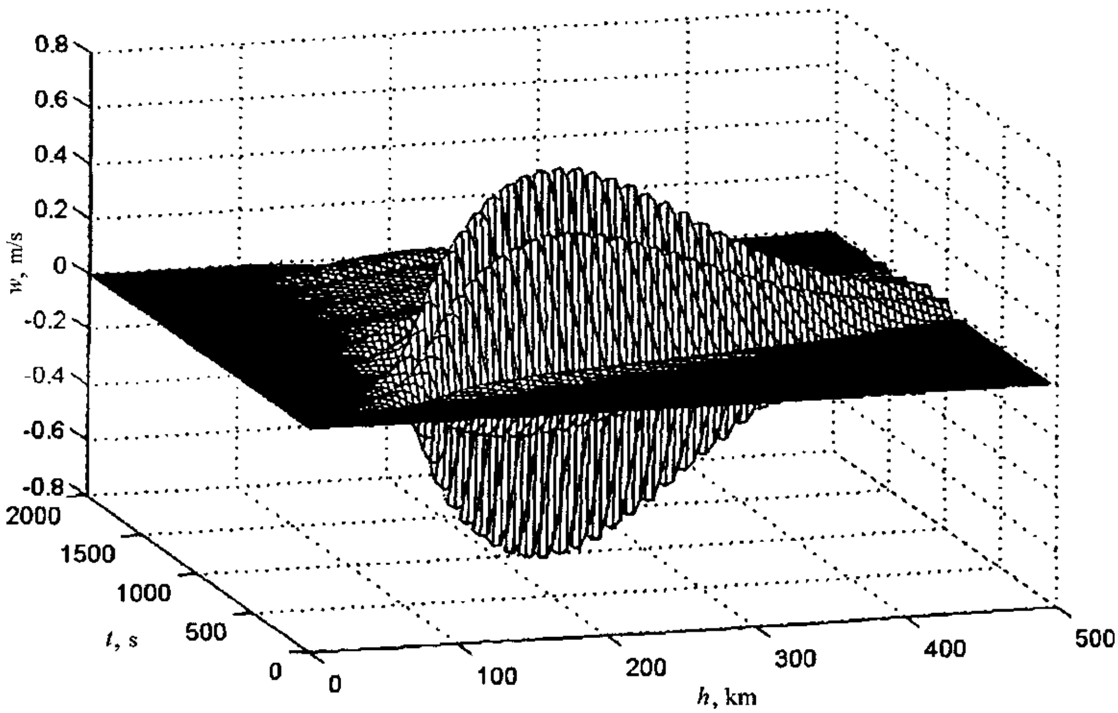

Fig. 1

Vertical component of air particle velocity of motion above the source.

The initial conditions for velocity, temperature and density disturbance were taken to be zero. A boundary condition for the velocity at the lower boundary will be the adhesion. The values of the sought-for quantities at the upper boundary are obtained by first-order extrapolation, i. e., the gradient of the quantities must be constant across the boundary. This allows the wave to cross the upper boundary of the domain of calculation without significant reflection. But for temperature, this condition excludes thermal conductivity through the upper boundary, therefore, the second derivative must be constant for it. For instance, for density at the upper boundary

$$\rho'^n_K = 2\rho'^n_{K-1} - \rho'^n_{K-2}.$$

RESULTS OF NUMERICAL SIMULATION

We solved numerically set of equations (4), which describes the model problem on a uniform grid with a step of 5 km in height and a step of 0.1 s in time. The numerical integration domain was set on a vertical coordinate from the Earth's surface to the height of 500 km. Actual profiles of the background density and temperature of the atmosphere in the model MSISE-90 (http://nssdc.gsfc.nasa.gov/space/model/models/msis.html) were used. After many checks and testings, the values $\nu = \mu = 0.02$ were chosen for the numerical diffusion and antidiffusion coefficients. The average value of the resistance coefficient was taken to be 0.0001.

The results obtained show that the developed numerical method is absolutely stable for the strongest pulse disturbances on the Earth. The results indicate generation of acoustic–gravity waves in the atmosphere by pulsed terrestrial sources. Figure 1 represents the air particle velocity against height and time. Here, the source parameters are $D_0 = 1.5$ m and $\Delta t = 0.2$ s, the maximum velocity of motion of the Earth's surface being 5.8 m/s. As the plot shows, wave periods range from 4 to 5 minutes. A 5-min period corresponds to the Brunt-Vaisala frequency of the atmosphere [1]. The temperature disturbance for such a source is shown in Fig. 2. A preliminary comparison of our model with the data in [2, 8, 12] obtained through observations demonstrates their qualitative agreement.

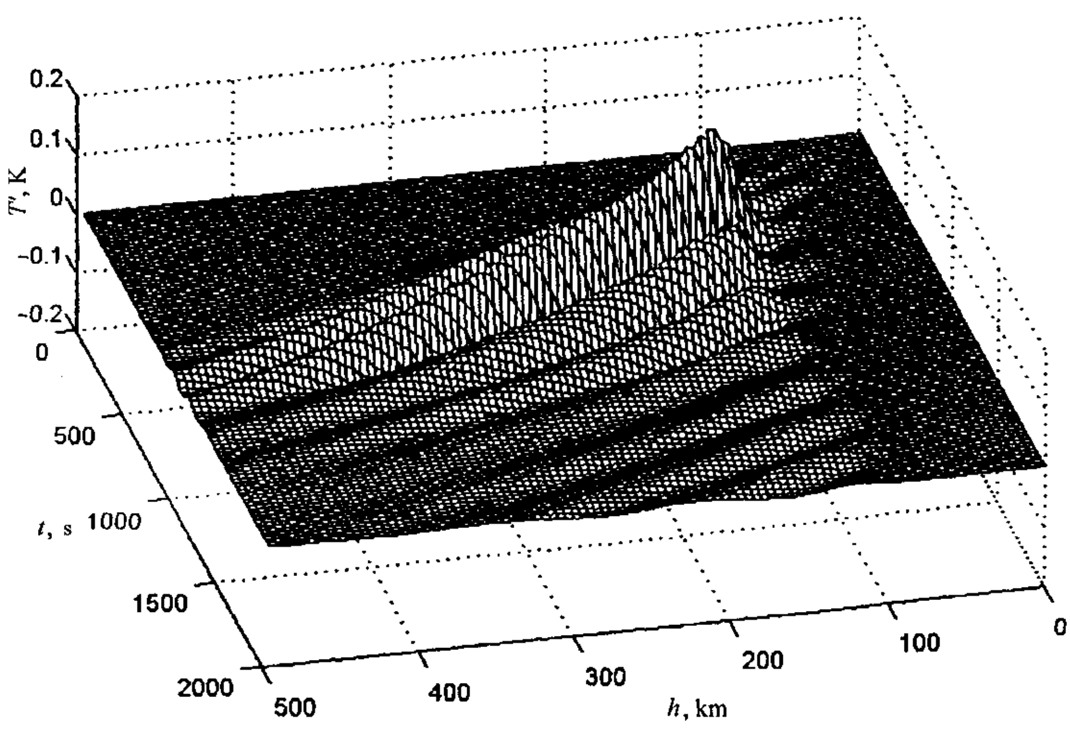

Fig. 2

Air temperature disturbances above the source.

The numerical algorithm helps us simulate generation and propagation of acoustic–gravity waves using various functions of the source. As the main problems of numerical solution of such equations are associated with a vertical transport, this method can successfully be employed in the two- and three-dimensional cases.

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