ISSN 0027-1349, Moscow University Physics Bulletin, 2017, Vol. 72, No. 3, pp. 224–235. © Allerton Press, Inc., 2017.
Original Russian Text © A.V. Belinsky, 2017, published in Vestnik Moskovskogo Universiteta, Seriya 3: Fizika, Astronomiya, 2017, No. 3, pp. 21–32.

OPTICS AND SPECTROSCOPY. LASER PHYSICS (REVIEW)

Light Interference and the Absence of Definite Values of Measured Quantities a Priori

A. V. Belinsky
Department of Physics, Moscow State University, Moscow, 119991 Russia
e-mail: belinsky@inbox.ru
Received June 25, 2016; in final form, September 6, 2016

Abstracts—The peculiarities of the behavior of photons in various experimental situations are considered. Variants of double-beam interference of a single photon and the possibility of forming a standing wave by this photon are analyzed; three-beam interference is also discussed. The effects observed in this case prove the absence of particular values of the measured quantities up to the moment of measurement; in the latter case, the number of photons in the field does not a priori have a certain value, in spite of the fact that the system is in the energy state. An experiment is also considered that proves the absence of a certain phase and the phase difference of photons in the Fock state, which makes it possible to treat different types of theories of hidden parameters, including nonlocal ones, more critically.

Keywords: interpretation of quantum theory, interference of a single photon, quantum measurements.
DOI: 10.3103/S0027134917030031

INTRODUCTION

Quantum measurements, in contrast to conventional measurements, have the characteristic property that prior to the measurement a physical quantity does not a priori have any specific magnitude, unless it is in the proper state of the measured quantity. This property, rather than the probabilistic nature of the measurement results, distinguishes quantum theory into an independent part of modern science; otherwise, it would simply be a subsection of statistical physics. This property is in full accordance with the Copenhagen interpretation of quantum theory. We consider specific experimental situations that support this condition below. The following issues should be borne in mind.

Many quantum effects that do not have classical analogs are paradoxical and cannot be interpreted from the point of view of macroscopic "common sense." The latter is usually quoted to show its inconsistency to essentially nonclassical phenomena. Researchers also call this "local realism." It is local, because models are used that are bound by space–time constraints that we usually observe in the macroworld. This is realism, because it is believed that, as in classical physics, physical quantities measured in experiments have quite definite values up to the moment of measurement.

The first doubts about the adequacy of local realism were generated by the interference effects of single photons in a Young double-slit interferometer [1] and Michelson and Mach–Zehnder dual-beam interferometers (see, for example, [2, 3] and citations therein). In these experiments, an indivisible quantum is present simultaneously in two channels and interferes with itself; until the moment of recording, its specific location is not determined.

Further, the effect of three-beam interference [4] proves the absence of a definite number of photons in the electromagnetic field a priori, that is, before measuring their number. Experiments to verify the inequalities of Bell [5], who formalized the Einstein–Podolsky–Rosen paradox [6] (including the latest ones [7, 8]), reliably disproved the local theory of hidden parameters, while its complete inadequacy was obvious from the time of the first tests [9–11] (see also [3, 12, 13]). The hypothesis of locality in this case assumes that two observers recording a pair of correlated particles, each following their own, are not connected in any way, and the readings of the measuring instruments of one do not affect the readings of the other (see, for example, [14, 15]). However, the validity of this assumption in the framework of experiments to test Bell's theorem is impossible to prove.

Thus, the only issue of the supporters of "realism," and, in fact, the reduction of the quantum theory to ordinary classical statistical physics is nonlocality as an unknown interaction of a mysterious nature that is not subject to either spatial or temporal (within the light cone) constraints. Clearly, an argument in favor of such views is the experimentally proven phenomenon of quantum nonlocality, not only for a pair or

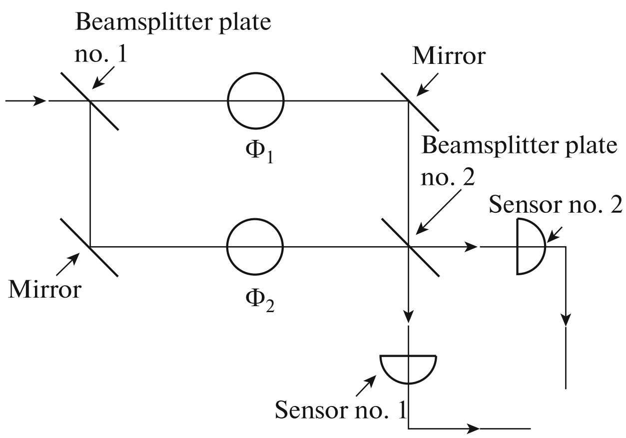

Fig. 1. The circuit of the Mach–Zehnder interferometer. The probability of photocounts at detectors is described by the harmonic function $P_{1,2} = (1/2)[1 \pm \cos(\Phi_1 - \Phi_2)]$, where Φ_1 and Φ_2 are the phase delays in the arms of the interferometer and the sign in this expression depends on which detector is recording.

more entangled particles, but also for a single photon [16–18].

In addition, the result of any experiment with quantum systems can be calculated using a computer, of course, in a probabilistic sense. The computer operates with specific values of the measured quantities that were completely determined before the moment of measurement (as in classical statistical physics). Therefore, it is quite difficult to absolutely refute strictly "nonlocal realism," or the nonlocal theory of hidden parameters, which is the same thing.

On the other hand, any physicist who deals with specific quantum computations will never believe, based on his experience and intuition, in "nonlocal realism." Therefore, to refute this concept, researchers went on to develop new experimental circuits that would increase the absurdity of models built on the basis of various types of nonlocal theories of hidden parameters even more. After all, how, for example, could the effects of double-beam interference be explained, considering the experiments to verify "deferred choice," or three-beam interference [4] within the framework of "nonlocal realism?" Only nonlocal "leaps" of photons between optical channels separated from each other, even through impenetrable walls, could explain this data [19].

A significant step towards testing "nononlocal realism" was made in [20–27]. These offered objective criteria (in the form of mathematical inequalities), which enable one, in particular, to refute one of the types of nonlocal theory that allows a nonlocal connection of measuring devices that record pairs of quantum particles in an entangled polarization state. The corresponding experiment, carried out by Zeilinger et al. [23], refuted this type of "nonlocal realism," although with the assumption that the Malus law is observed. However, to doubt the validity of the latter means to further increase the degree of absurdity of a nonlocal-realistic interpretation.

Nevertheless, a stronger form of nonlocality, namely, the nonlocal connection of the results of measurements of recorders remote from one another, rather than the measuring devices themselves, cannot be refuted in this way. However, the absurdity of "nonlocal realism" can be increased by the experiment described at the end of this review, combining both the effect of suppression of the mutual correlation of photons [28] and the preparation of quantum squeezed states (see, for example, [29] and citations therein). Let us first consider simpler experiments that prove the absence of certain values of the measured quantities without involving nonlocal theories of hidden parameters.

1. TWO-BEAM INTERFERENCE

In conventional two-beam interferometers, for example, a Mach–Zehnder interferometer (Fig. 1), the harmonic dependence of the light intensity I at the output of the phase difference in the channels $\Phi_2 - \Phi_1$ occurs as a result of the addition of complex amplitudes of two interfering beams (see, for example, [4]):

$$I \propto |a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + a_1 a_2^* + a_1^* a_2 = |a_1|^2 + |a_2|^2 + 2|a_1 a_2|\cos(\Phi_2 - \Phi_1). \tag{1}$$

The interference terms $a_1 a_2^*$ and $a_1^* a_2$ are the product of two amplitudes, one of which is a complex conjugate. Such interference can be called second-order interference (in terms of amplitude). In the quantum description of two-mode interference, these terms will be the operators $\hat{a}_1 \hat{a}_2^+$ and $\hat{a}_1^+ \hat{a}_2$, because the interferometer is a linear device; hence, noncommuting operators do not multiply anywhere else and their noncommutativity does not affect the result in any way. This means that the classical description of a linear system differs from the quantum one only in that the complex amplitudes become operators in the Heisenberg representation in the quantum description, and the quantum specificity is manifested only when the system is averaged over the quantum state.

Let us consider a Mach–Zehnder interferometer to which single photons in the Fock state are fed (Fig. 1). First, we remove the second beam splitter located in front of the photodetectors. We turn on the detectors, which begin to record single photocounts in either one or the other channel with an equal probability of 1/2.

What happens after we return the second beam splitter to its place? The probability of photocounts on detectors is now described by a harmonic function (see, for example, [3]):

$$P_{1,2} = 1/2[1 \pm \cos(\Phi_1 - \Phi_2)], \tag{2}$$

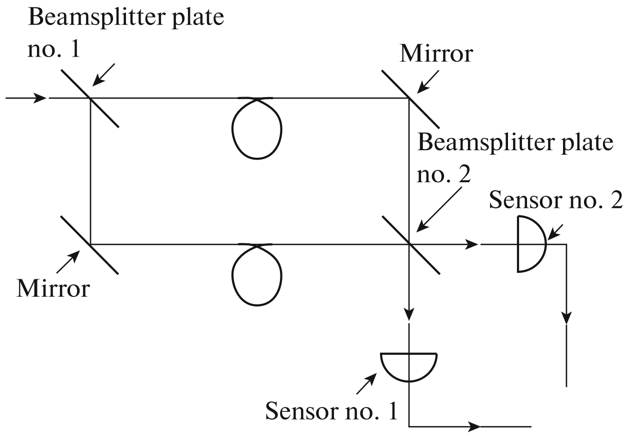

Fig. 2. The circuit of the Mach–Zehnder interferometer with identical nonlinear fibers in the channels.

where Φ_1 and Φ_2 are the phase delays in the arms of the interferometer. The sign in this expression depends on which detector is recording. This harmonic function cannot be represented as a sum of probabilities

$$P_{1,2}(\Phi_1, \Phi_2) \neq P(\Phi_1) + P(\Phi_2). \quad (3)$$

Consequently, after the first beam splitter, a photon is present in both arms of the interferometer simultaneously; although, in the first stage of the experiment, it was recorded only in one. This is because the state vector of a quantum system is reduced only at the time of recording of a photon, while before that, it is in both channels.

The same result can be proved in another way. If the phase difference is $0 + 2\pi m$ or $\pi + 2\pi m$, single photons occur only in one of the detectors. The probability of the appearance of photons on the second detector is zero. We close one of the arms of the interferometer. Photocounts will appear on the photodetector that was "silent" until this moment. This means that before the closing, the photon was present in this arm in each test; otherwise, the probability of the appearance of photocounts on the considered photodetector would not be zero. The presence of a photon in each test in the other arm of the interferometer is proved similarly. Consequently, the result of measuring the location of the photon (in which arm is the photon?) with the removed second beam splitter before the measurement is not determined, since the photon is in both shoulders at once [3].

There is another rigorous proof of the presence of a photon in both arms of the interferometer simultaneously. We place two identical nonlinear media that have cubic nonlinearity instead of, or as, phase delays; phase self-modulation (PSM) occurs in these media, that is, a change in the refractive index of the media under the effect of light in them. This can occur, for example, using quartz fibers (Fig. 2). A photon passing through them must acquire an additional phase foray, which will inevitably affect the result of the interference.

Suppose that in the absence of radiation, the phase forays in the arms were identical. Then, by sending a single photon to the interferometer, we have two alternatives: either the photon passes through only one arm and the phase difference changes due to the nonlinear phase foray in this arm, or the photon passes through both arms, the nonlinear phase forays in which remain the same, so that the phase difference does not change. In the latter case, the photon will appear only in one of the outputs of the interferometer.

The input monochromatic mode in the Fock state $|1\rangle$ is described below by the photon annihilation operator of $\hat{a}_1$ and the vacuum mode $|0\rangle$ at the second input is described by the operator $\hat{a}_0$. After the first 50% beam splitter, we also consider two modes described by operators $\hat{a}_2$ and $\hat{a}_3$ in the Heisenberg representation, that is,

$$\hat{a}_2 = \frac{\hat{a}_1 + \hat{a}_0}{\sqrt{2}}, \quad \hat{a}_3 = \frac{\hat{a}_1 - \hat{a}_0}{\sqrt{2}}. \quad (4)$$

Next, we take the action of the Kerr nonlinearity into account. The stable transverse intensity distribution in quartz fibers can be regarded as a mode of radiation, and the four-photon process itself can be described by a single-mode Hamiltonian (see, for example, [13] and citations therein)

$$\hat{H} = \frac{\hbar}{2}\chi^{(3)}\hat{a}^+\hat{a}^+\hat{a}\hat{a}, \quad (5)$$

where $\chi^{(3)}$ is the cubic nonlinearity coefficient, normalized by the number of photons. We assume the nonlinear response to be instantaneous.

The corresponding evolution operator of the quantum state in the Schrödinger representation is

$$\hat{U} = \hat{I}\exp\left(-i\frac{\overline{\chi}}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a}\right) = \hat{I}\exp\left(-i\frac{\overline{\chi}}{2}\hat{n}(\hat{n}-1)\right), \quad (6)$$

where $\overline{\chi} = \chi^{(3)}t$; the evolution time t is related to the fiber length as $l = vt$, v is the mode propagation speed in the fiber, and $\hat{n}(t)$ is the photon number operator.

In the Heisenberg representation, the photon annihilation operator of the field mode obeys equation $i\hbar\frac{d\hat{a}}{dt} = [\hat{a}, \hat{H}]$, whence $\hat{a}(t) = e^{-i\overline{\chi}\hat{a}^+(0)\hat{a}(0)}\hat{a}(0)$, and in our case,

$$\hat{a}_2' = e^{-i\overline{\chi}\hat{a}_2^+\hat{a}_2}\hat{a}_2, \quad \hat{a}_3' = e^{-i\overline{\chi}\hat{a}_3^+\hat{a}_3}\hat{a}_3. \quad (7)$$

Accordingly, the two output modes of the interferometer are

$$\hat{a}_2' = \frac{\hat{a}_2' - \hat{a}_3'}{\sqrt{2}}, \quad \hat{a}_1' = \frac{\hat{a}_2' + \hat{a}_3'}{\sqrt{2}}. \quad (8)$$

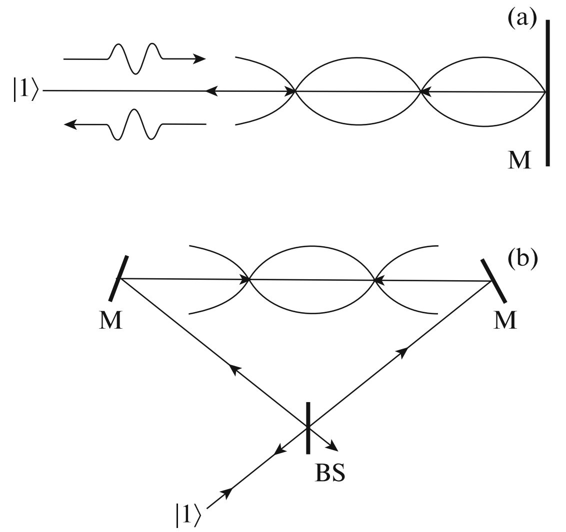

Fig. 3. A standing wave formed by a "single-photon wave packet" (a) under normal reflection from mirror M and (b) the photon forms a standing wave symmetric with respect to reflecting mirrors M, using a 50% beam splitter (BS).

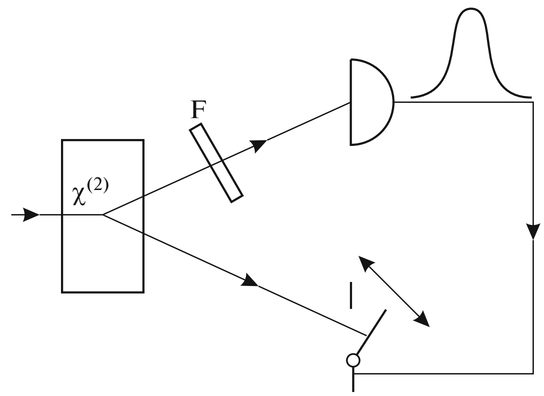

Fig. 4. The circuit for the preparation of a single-photon state: one of the photons of the correlated pair (the upper one) is detected by a detector, the electronic signal of which opens the shutter that passes the second photon of the correlated photon pair, which, of course, must enter the shutter with a delay. The shutter then closes quickly again. The bandwidth of the prepared photon is determined by the bandwidth of spectral filter F, since the sum of the frequencies of the photon pair is strictly the same as the frequency of monochromatic pumping.

Let us find the average values of the photon numbers at the outputs of the interferometer, namely,

$$\langle \hat{n}_0 \rangle = \langle \hat{a}_0^{'+} \hat{a}_0^{'} \rangle = 0, \quad \langle \hat{n}_1 \rangle = \langle \hat{a}_1^{'+} \hat{a}_1^{'} \rangle = 1. \tag{9}$$

Thus, we observe interference with a zero phase difference, so that the photon is present in both channels simultaneously.

2. A STANDING WAVE

Strictly speaking, a single photon in a planar monochromatic mode has an infinite length. What prevents it from interfering with itself, for example, after a normal fall and reflection from an ideal reflecting mirror, as in Fig. 3a [30]?

As indicated above, the classical description of the linear system until the moment of averaging over the quantum state of the system does not differ from the quantum one. Thus, in the case of plane fronts, a single photon will be "smeared" along a stationary standing wave. Does this mean that it "flies" to a flat mirror and then, reflected, "flies" back? This is obviously not the case, because it is present in the entire standing wave. Therefore, measuring the location of a photon by a photodetector just visualizes it, as it did not have a specific coordinate before measurement. We again come to the conclusion that there is no definite value of the measured value until the moment of measurement.

A standing wave can also be formed in a resonator with the only proviso being that in a resonator, a standing wave is formed not by a plane mode, but by a mode (or modes) of the resonator.

It is impossible to record a photon with a quadratic detector that reacts to the square of the amplitude of the electric field at the nodes of a standing wave, since it is zero. The photon has a magnetic component, as well as its electric component. The presence of the magnetic component in the nodes of a standing wave can be detected as follows. If a standing wave is pierced with an electron beam, electrons do not deviate at the nodes, because there is no impulse action of the photon on them. One can also try to observe an X-ray standing wave with an electron microscope. A decrease in the deviation of electrons in the nodes should be noticeable. However, if the electrons are relativistic, they deviate even at the nodes of the standing wave due to the Lorentz force that acts in the presence of the magnetic component. This would be an interesting experiment.

Certainly, there are no monochromatic modes in the universe. Nevertheless, it is always possible to form a narrow-band "one-photon wave packet" that will have a certain coherence length that is required to observe interference for a certain period. The standing wave in this case is no longer single-mode and static, which, however, is a simplified idealization. Such states, that is, single-photon wave packets, are prepared using a special device that includes a nonlinear parametric light converter. The following section is devoted to the description of the circuit of this device (Fig. 4).

3. PREPARATION OF A SINGLE-PHOTON STATE

An excellent method for preparing a single-photon state using parametric light scattering was first

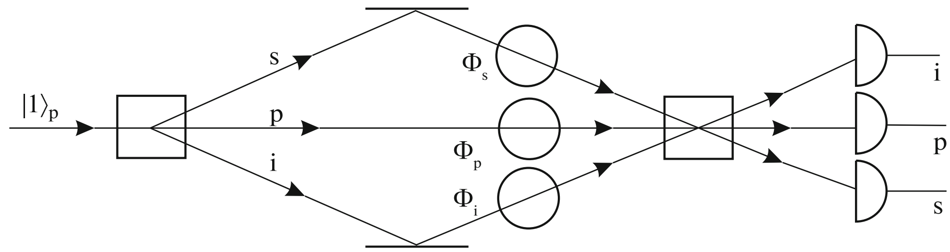

Fig. 5. The circuit of the interference experiment that proves the a priori nonexistence of a certain number of photons in the field between two nonlinear crystals, as represented in the form of squares. Single photons $|1\rangle_p$ at the frequency of ω_p are fed to the input. The probability of photocounts at detectors i or s is proportional to $1 + \cos(\Phi_s + \Phi_i - \Phi_p)$, which indicates the simultaneous presence of radiation in all three channels; however, the energy of one input photon is sufficient only for half the energy of three photons.

proposed by Klyshko [31]. The circuit is shown in Fig. 4. Hong and Mandel carried out the first experiment [32].

Parametric light transformation is the process of production of photon pairs in a transparent medium with quadratic nonlinearity under the action of monochromatic pumping with a frequency of ω_p. Two radiation beams are generated: signal (s) and idler (i), whose frequencies are related by $\omega_p = \omega_s + \omega_i$.

The power of the pump laser is selected to be low so that the production of photon pairs in the course of parametric scattering is quite rare, otherwise they will be superimposed on each other. To cut off the pairs that are "missed" by the detector and all types of extraneous flare, a shutter is placed in the second channel of the circuit in Fig. 4 that transmits only the photon of the pair detected by the detector. After the shutter, we have a single photon at a certain moment (more precisely, a gap) of time of the photocount pulse formation and in a certain region of space (immediately behind the shutter). We collimate it and direct it to the input of the interference circuits that were considered previously.

Spectral filter F is set in order to form a single-photon wave packet of the required coherence length that is the reciprocal filter transmission width. The fact is that the sum of the photon frequencies of the correlated pair is strictly the same as the frequency of monochromatic pumping; hence, the widths of the bands of the signal and idler beams are equal. Thus, by decreasing the width of spectral filter F, we obtain an increase in the coherence length of the prepared single-photon wave packet.

4. THREE-BEAM INTERFERENCE

Along with conventional two-beam interferometers, in particular, those considered above, there are also interferometers in which a set of beams with different amplitudes are added. These are, for example, Fabry–Pérot interferometers with plane parallel resonators or mirrors, or multi-beam interferometers with mirrors of another shape. Their advantage, with respect to the two-beam type, is that they are more sensitive to phase delays. Here, we consider a three-beam interferometer that is not so much of practical as of heuristic interest, because it enables the assessment of specific and at the same time fundamental features of quantum theory in general and the theory of quantum measurements in particular. The circuit is presented in Fig. 5 [4, 30].

A light beam with frequency ω_p in a transparent nonlinear crystal generates two beams of radiation in the course of parametric scattering: a signal beam (s) and an idler beam (i). The interaction over space is nondegenerate: the beams are noncollinear. Since the efficiency of the parametric transformation is low, of the order of 10^{-8} to 10^{-7}, the main part of the radiation passes through a transparent crystal, at the output of which three beams, in the simplest case, three modes (p, s, and i), are formed. Further, adjustable phase shifts Φ_p, Φ_s, and Φ_i are introduced into all three field components, after which they again interact in the second, exactly the same nonlinear crystal. This carries out an inverse transformation of the signal and idler beams into radiation at the pump frequency ω_p and the direct conversion of the pumping transmitted the first crystal. Detectors at the output of the optical circuit detect the photocounts of all three beams.

Let us return to parametric scattering. With a small probability (on the order of 10^{-8} to 10^{-7}), the pump photon (p) disappears, which is described by the photon annihilation operator $\hat{a}_p$, and a signal (s) and idler (i) photons are generated, as described, respectively, by the creation operators $\hat{a}_s^+$ and $\hat{a}_i^+$. Thus, if we act on the left by one-photon pumping state $|1\rangle_p$ and the vacuum states of the signal and idler modes $|0\rangle_s$ and $|0\rangle_i$ by the operator $\hat{a}_p\hat{a}_s^+\hat{a}_i^+$, we obtain a correlated photon pair

$$\hat{a}_p\hat{a}_s^+\hat{a}_i^+|1\rangle_p|0\rangle_s|0\rangle_i = |0\rangle_p|1\rangle_s|1\rangle_i. \quad (10)$$

Here, we mean three plane monochromatic radiation modes.

These simple considerations explain the structure of the three-mode effective interaction Hamiltonian of the parametric scattering process, that is,

$$\hat{H} \propto \frac{i\hbar\chi^{(2)}}{2}\hat{a}_{\mathrm{p}}\hat{a}_{\mathrm{s}}^{+}\hat{a}_{\mathrm{i}}^{+} + \mathrm{H.c.}, \tag{11}$$

where $\chi^{(2)}$ is a quadratic nonlinearity, and the hermitian adjoint operator

$$\mathrm{H.c.} = -\frac{i\hbar\chi^{(2)}}{2}\hat{a}_{\mathrm{p}}^{+}\hat{a}_{\mathrm{s}}\hat{a}_{\mathrm{i}}^{+} \tag{12}$$

describes the process that is the inverse of parametric scattering, namely, the creation of a pump photon with the simultaneous disappearance of signal and idler photons, which is also possible.

The effective three-mode Hamiltonian of parametric scattering interaction (Eq. (11)) indicates that in the case of a single-photon pump state, the exact solution of the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \tag{13}$$

should be sought in the form of

$$|\psi(t)\rangle = \alpha(t)|1\rangle_{\mathrm{p}}|0\rangle_{\mathrm{s}}|0\rangle_{\mathrm{i}} + \beta(t)|0\rangle_{\mathrm{p}}|1\rangle_{\mathrm{s}}|1\rangle_{\mathrm{i}}. \tag{14}$$

In fact, if at the crystal input, one pump photon $|1\rangle_{\mathrm{p}}$ and the vacuum at the signal and idler modes $|0\rangle_{\mathrm{s}}$ and $|0\rangle_{\mathrm{i}}$ occur, then either this photon "falls apart" into the signal and idler ($|1\rangle_{\mathrm{s}}$ and $|1\rangle_{\mathrm{i}}$) photons and a vacuum $|0\rangle_{\mathrm{p}}$ occurs in the pump, or everything remains the same. There is no other alternative. In this case, $\alpha(0) = 1$, and $\beta(0) = 0$. The same considerations apply to the inverse transformation in the second crystal with the only difference that there, $\alpha(t)$ and $\beta(t)$ may have other initial conditions, but in any case,

$$|\alpha(t)|^2 + |\beta(t)|^2 = 1, \tag{15}$$

otherwise, the normalization condition for the state vector (Eq. (14))

$$\langle\psi(t)|\psi(t)\rangle = 1$$

is not satisfied. We call attention to the fact that the state vector (14) is not factorized,

$$|\psi(t)\rangle \neq |\psi(t)\rangle_{\mathrm{p}}|\psi(t)\rangle_{\mathrm{s}}|\psi(t)\rangle_{\mathrm{i}}, \tag{16}$$

that is, it is a typically entangled state.

To verify the validity of solution (14), we substitute it into the Schrödinger equation (13)

$$i\hbar(\dot{\alpha}|100\rangle + \dot{\beta}|011\rangle) = i\hbar\frac{\chi^{(2)}}{2}(\alpha|011\rangle - \beta|100\rangle), \tag{17}$$

where $|100\rangle = |1\rangle_{\mathrm{p}}|0\rangle_{\mathrm{s}}|0\rangle_{\mathrm{i}}$, and $|011\rangle = |0\rangle_{\mathrm{p}}|1\rangle_{\mathrm{s}}|1\rangle_{\mathrm{i}}$, from where

$$\begin{cases} \dot{\alpha} = -\dfrac{\chi^{(2)}}{2}\beta, \\ \dot{\beta} = \dfrac{\chi^{(2)}}{2}\alpha. \end{cases} \tag{18}$$

It is convenient to represent the solution of this linear system of differential equations in a matrix form

$$\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} = \begin{pmatrix}\cos\Gamma & -\sin\Gamma\\ \sin\Gamma & \cos\Gamma\end{pmatrix}\begin{pmatrix}\alpha(0)\\ \beta(0)\end{pmatrix}, \tag{19}$$

where $\Gamma = \dfrac{\chi^{(2)}}{2}t$ and the interaction time is determined by the time of flight of a photon through a nonlinear crystal. It is assumed here that the phase-matching condition is satisfied for the selected directions of propagation of the mode in the crystal; that is, all three modes propagate synchronously when the law of conservation of momentum is fulfilled.

The matrix of the transformation of the field state in a crystal is thus

$$D(\Gamma) = \begin{pmatrix}\cos\Gamma & -\sin\Gamma\\ \sin\Gamma & \cos\Gamma\end{pmatrix}. \tag{20}$$

We now turn to the description of phase delays in modes. The phase-shift operator of the mode is (see, for example, [13])

$$\hat{U}_{\theta} = \exp(-i\theta\hat{n}),$$

where $\hat{n} = \hat{a}^{+}\hat{a}$ is the photon number operator in the mode and θ is the phase delay. Since we have either one or zero photons in each mode, the operator $\hat{n}$ for its own state is transformed into the C-number, respectively, 1 or 0. Consequently, the $|100\rangle$ component must be multiplied by $e^{-i\Phi_{\mathrm{p}}}$. Similarly, the second component $|011\rangle$ is multiplied by $e^{-i(\Phi_{\mathrm{s}}+\Phi_{\mathrm{i}})}$. In this case, the transformation matrix is diagonal, that is,

$$D(\Phi) = \begin{pmatrix} e^{-i\Phi_{\mathrm{p}}} & 0\\ 0 & e^{-i(\Phi_{\mathrm{s}}+\Phi_{\mathrm{i}})}\end{pmatrix}. \tag{21}$$

Thus, the overall effect of the optical system of the interferometer in Fig. 5 is described by the matrix

$$D = D(\Gamma_2)D(\Phi)D(\Gamma_1), \tag{22}$$

where the indices 1 and 2 refer to the first and second nonlinear crystals, respectively.

Since in the initial state,

$$\begin{pmatrix}\alpha(0)\\ \beta(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}. \tag{23}$$

at the output of the second nonlinear crystal, state (14) will occur with

$$\alpha(t) = e^{-i\Phi_p}\cos\Gamma_1\cos\Gamma_2 - e^{-i(\Phi_s+\Phi_i)}\sin\Gamma_1\sin\Gamma_2, \quad (24)$$

$$\beta(t) = e^{-i\Phi_p}\cos\Gamma_1\sin\Gamma_2 + e^{-i(\Phi_s+\Phi_i)}\sin\Gamma_1\cos\Gamma_2. \quad (25)$$

The probabilities of the operation of ideal detectors in the channels are

$$P_p = \langle\psi(t)|\hat{a}_p^+\hat{a}_p|\psi(t)\rangle = \langle\psi(t)|\hat{n}_p|\psi(t)\rangle = |\alpha(t)|^2, \quad (26)$$

$$P_s = P_i = \langle\psi(t)|\hat{a}_s^+\hat{a}_s|\psi(t)\rangle = \langle\psi(t)|\hat{a}_i^+\hat{a}_i|\psi(t)\rangle = |\beta(t)|^2. \quad (27)$$

Thus, according to Eqs. (24) and (25),

$$P_p = C_p(1 + v_p\cos\Phi), \quad (28)$$

$$P_s = P_i = C_s(1 + v_s\cos\Phi), \quad (29)$$

where the so-called *visibility of interference*

$$v_p = \frac{2\sin\Gamma_1\sin\Gamma_2\cos\Gamma_1\cos\Gamma_2}{\sin^2\Gamma_1\sin^2\Gamma_2 + \cos^2\Gamma_1\cos^2\Gamma_2} = \frac{2}{\tan\Gamma_1\tan\Gamma_2 + \cot\Gamma_1\cot\Gamma_2}, \quad (30)$$

$$v_s = \frac{2\sin\Gamma_1\sin\Gamma_2\cos\Gamma_1\cos\Gamma_2}{\sin^2\Gamma_1\cos^2\Gamma_2 + \cos^2\Gamma_1\sin^2\Gamma_2} = \frac{2}{\tan\Gamma_1\cot\Gamma_2 + \cot\Gamma_1\tan\Gamma_2}, \quad (31)$$

phase of interference

$$\Phi = \Phi_p - \Phi_s - \Phi_i, \quad (32)$$

and coefficients C_p and C_s are

$$C_p = \sin^2\Gamma_1\sin^2\Gamma_2 + \cos^2\Gamma_1\cos^2\Gamma_2, \quad (33)$$

$$C_s = \sin^2\Gamma_1\cos^2\Gamma_2 + \cos^2\Gamma_1\sin^2\Gamma_2. \quad (34)$$

Thus, the probabilities of detecting photons in channels are described by the harmonic dependences (28) and (29) on phase (32), which includes phase delays in all three channels. Therefore, they can be called interference dependencies, and the phenomenon itself is *a third-order interference* or *three-beam interference*.

The maximum contrast of the interference pattern, that is, the maximum difference between the interference minimum and the maximum, occurs at a single visibility v. In the s and i channels this is rather easy to achieve: it is sufficient to take identical nonlinear crystals with $\Gamma_1 = \Gamma_2$. Then, according to Eq. (31), $v_s = 1$. In the pump channel p, this procedure is more complicated, because $v_p = 1$ for $\Gamma_1 = \Gamma_2 = \pi/4$, which requires a very high efficiency of the parametric interaction; this is hardly possible with single-photon pumping.

To observe the interference effect, in principle, one detector is sufficient, for example, in the s channel. In this case, a single visibility will also exist in the case of a small parametric transformation efficiency, that is, for any $\Gamma_1 = \Gamma_2$, including $\Gamma_1 = \Gamma_2 \ll 1$. Therefore, the experiment is quite feasible. However, it is necessary to prepare a single-photon pumping state. An ideal laser will not help here, because even at an average intensity of one photon during observation time T, the number of recorded photons from one experiment to another fluctuates according to the Poisson law from zero to four and higher. Therefore, we again need to use the circuit given in Fig. 4.

Since the production of photon pairs is quite rare and the production of secondary pairs in nonlinear crystals of a three-beam interferometer in Fig. 5 is even rarer, patience is required to record the interference dependence of Eq. (29). It is, however, possible in principle. The first experiments to observe the interference of single photons on two slits sometimes lasted for 3 months [33], because circuits similar to that in Fig. 4 did not exist and single photons were taken from a very weak heat source so that the probability of occurrence of two photons instead of one was negligible. In addition, the experimental cascade parametric preparation of photon triples [34] leaves no doubt about the possibility of carrying out the proposed experiment.

Previously, experimenters [35] observed three-beam interference with ordinary laser light at the input rather than with single photons. Harmonic interference dependence (29) was obtained, however, with a visibility of $v_s = v_i$ somewhat smaller than unity. It is very difficult to avoid all kinds of noise in the "zeros" of interference, where $P_s = P_i = 0$. These "zeros" are interesting from the point of view of the interpretation of the experiment.

Let us try to interpret result (29) in the framework of the visual model with a priori a certain number of photons in the three-mode field between nonlinear crystals (Fig. 5). A priori in this case means until the moment of triggering of any of the detectors. For simplicity, their quantum efficiency η is set to unity.

In the first test series, we remove the second nonlinear crystal, which corresponds to $\Gamma_2 = 0$. In this case, the phase delays in the channels do not affect the detection results and photocounts occur either in both channels s and i simultaneously or single counts in channel p. The latter, of course, occur much more frequently. However, there will never be three photocounts simultaneously in all three channels, since the photon energy at the input is two times too low for this. Such a pattern agrees with the naive trivial assumption that at the output of the first nonlinear crystal alternately one pumping photon $|1\rangle_p$ and the pair of signal and idler photons $|1\rangle_s|1\rangle_i$ supposedly occur.

Quantum state (14) is interesting in this case because the three-mode field has a strictly defined energy $\hbar\omega_p$, although the total number of photons

$$n_{p+s+i} = n_p + n_s + n_i \quad (35)$$

at $\alpha\beta \neq 0$ does not have a precisely determined value,

$$\langle\psi(t)|\hat{n}_{p+s+i}|\psi(t)\rangle = |\alpha|^2 + 2|\beta|^2 = 1 + |\beta|^2, \quad (36)$$

$$\langle\psi(t)|\Delta\hat{n}^2_{p+s+i}|\psi(t)\rangle = |\alpha\beta|^2, \quad (37)$$

that is, the variance of the fluctuations of the total number of photons is nonzero. This still fully agrees with the simple and naive assumption that one or two photons appear in the field at once.

Let us now describe the second series of tests.

We set the second nonlinear crystal with $\Gamma_2 = \Gamma_1$ into the circuit in Fig. 5. In this case, all three phases Φ_p, Φ_s, and Φ_i affect the probabilities of photocounts according to Eqs. (28), (29), and (32).

Interference with a single visibility $v_s = v_i = 1$, described by equation

$$P_s = P_i \propto 1 + \cos\Phi, \quad (38)$$

shows that by changing the phase delay of any component of the field: Φ_s, Φ_p, or Φ_i, it is possible to completely suppress the photocounts at $\cos(\Phi_p - \Phi_s - \Phi_i) = -1$. This is the "zero" of interference or the interference minimum. In this case, photocounts, for example, in channel s will not be detected.

We close the light in the gap between the nonlinear crystals in channel *p* in the same way as we did with the Mach–Zehnder interferometer. Photoshots appear in the *s* channel because their probability ceases to be zero. Therefore, if there was no field in channel *p* in at least one test of the circuit with all three open channels, the probability of photocounts in the detector *s* would be nonzero. However, it is zero! Therefore, the field in the pumping channel *p* is present in *every* test. Similarly, by interrupting the light in other channels, we prove the simultaneous presence of a light field in each test in the channels of the pair of photons *s* and *i*. In other words, if for all open channels in any test, that is, in launching a single pump photon into the input, the field would be absent in at least one of the channels, then the probability of photocounts at detector s would be nonzero. Hence, the field is present in *every* test in all three channels *p*, *s*, and *i* between nonlinear crystals. However, the simultaneous presence of all three photons is impossible due to the law of conservation of energy: in fact, only one pumping photon ($\beta(0) = 0$) was fed to the interferometer input, the energy of which is half the total energy of all three photons.

There is, however, one weak link in the above arguments: in reality, as in the experiment mentioned above [8], it is hardly possible to achieve a single visibility (contrast) or absence of noise background. Hence, there are no interference "zeros." Nevertheless, the simultaneous presence of the field in all three channels is indicated by the cosine dependence of the probability $P_s = P_i$ (38) on the superposition of phases (32), confirmed experimentally [8]. In fact, if the field between nonlinear crystals (Fig. 5) contains one or two photons (as given by direct measurements) from test to test, then $P_s = P_i$ could be represented as a sum of two functions, one of which would depend only on Φ_p while the other depends on Φ_s and Φ_i. However, $\cos(\Phi_p - \Phi_s - \Phi_i)$ does not allow such a representation.

Therefore, the number of photons and, in a more general case, a measured value in general, does not have a definite value until the measurement, except for the situation where the object of measurement is in the proper state of the measured quantity. The superposition of single- and two-photon states in the circuit in Fig. 5 leads to their interference, which proves the a priori nonexistence of a certain number of quanta in the field before the moment of measurement. This raises serious concerns about the genuineness of the existence of photons in general.

As Klyshko wrote, "A photon is a photon if it is a recorded photon" [36]. The source and receiver must be viewed in a complex, without taking one from the other [37, p. 414]. In quantum theory, this requirement, as a rule, is fulfilled automatically: the quantum theory predicts only the results of measurements and measurements assume the presence of detectors. Therefore, detectors are always present in tasks. This means that we cannot say anything definite about what happens between the production and annihilation of a photon. Or, rather, there is no physical reality that cannot be measured. In this sense, a photon is not a physical reality until its recording.

5. ON THE UNCERTAINTY OF THE PHOTON PHASE IN FOCK STATES

It is well known that due to the Heisenberg uncertainty principle, the phase (or its cosine and sine, which are observable and described by Hermitian operators) in the Fock states with a certain number of photons is completely undefined. Is it in the form of a superposition of all its possible values, or does it have some unknown quantity? Here, we are faced again with the opposition of the Copenhagen interpretation of the nonlocal theory of hidden parameters. How can this uncertainty of the phase be interpreted by the nonlocal theory of hidden parameters? Only asserting that a photon in a state, for example, $|1\rangle$ still has a phase, but this phase nonlocally "adapts" to a specific experimental situation, as if it "knows" the entire subsequent history of conversion and measurement of a photon in advance. This explains not only the violation of Bell's inequalities, but also all possible interference quantum effects. Let us try to refute these ideas by assessing the effects of suppressing the correlation of photocounts [28] and the preparation of compressed states in the parametric scattering of light (see, for example, [29]).

The effect of suppressing the correlation of photocounts is an amazing and incomprehensible, in the

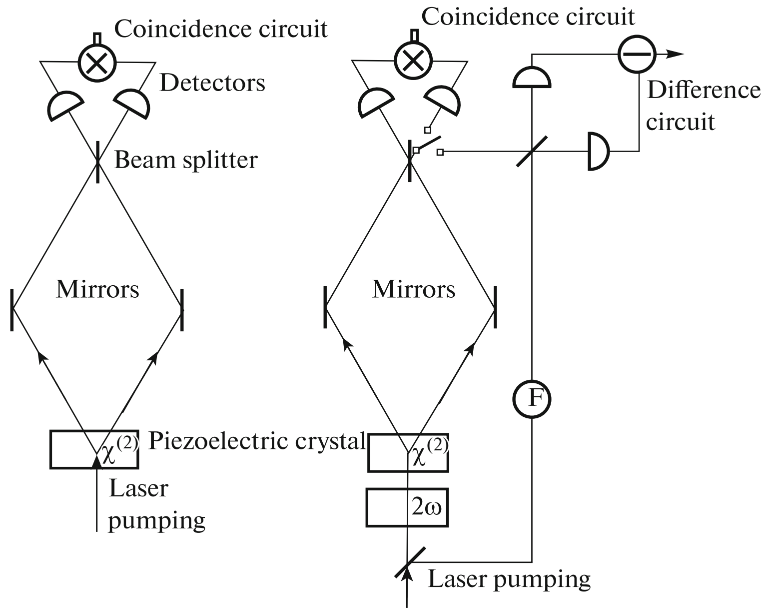

Fig. 6. A circuit for observing the effect of suppression of the mutual correlation of photons (left) and simultaneous recording of the compressed state (right). In a nonlinear crystal, usually a piezoelectric crystal, pairs of photons are generated that are directed to the beam splitter and are detected. The coincidence circuit records the simultaneous arrival of photons to both detectors (left). The probability of such events is zero. Using the circuit of balanced homodyne detection, fluctuations of the quadrature component of the field are simultaneously recorded (right). In this circuit, the radiation is directed by the operating mode switch. To match the mixed homodyne frequencies, the frequency of the laser radiation is doubled in front of the nonlinear crystal.

sense of its interpretation, phenomenon that demonstrates the specificity of quantum theory. It consists in the following. If a single photon is fed to one of the inputs of a 50% beam splitter, then it appears at one of the outputs with a probability of 1/2, thus exhibiting typically corpuscular properties. What happens if one feeds a photon to each of the beam splitter inputs at the same time? It would seem that there should be two photons at one of the outputs with probabilities of 1/4 or one of the photons at each output with a probability of 1/2. In fact, it does not work that way: the probability of the second event turns out to be zero, while the photons at the outputs appear only in pairs. How can we be sure of this? In the experiment in [28], signals from two detectors installed at beam-splitter outputs that operated in the single photocount mode were directed to a coincidence circuit (Fig. 6 (left)). To within the technical noise, the signal with the latter was zero.

Theoretically, this result can be described in the representation of both Heisenberg and Schrödinger. In the former, we introduce the photon annihilation operators describing two input plane monochromatic modes, $\hat{a}$ and $\hat{b}$. The output mode operators in this case are $\hat{c} = (\hat{a} + \hat{b})/\sqrt{2}$ and $\hat{d} = (\hat{a} - \hat{b})/\sqrt{2}$. Next, operators of photon numbers $\hat{n}_c = \hat{c}^+\hat{c}$ and $\hat{n}_d = \hat{d}^+\hat{d}$ are found; their correlation function $\langle \hat{n}_c\hat{n}_d \rangle$ is then determined by averaging over the initial state of $|1\rangle_a\,|1\rangle_b$. The result is ${}_a\langle 1|_b\langle 1|\hat{c}^+\hat{c}\hat{d}^+\hat{d}|1\rangle_b|1\rangle_a = 0$.

In the Schrödinger representation, we need to introduce a beam splitter matrix

$$B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} \tau & -\rho \\ \rho & \tau \end{pmatrix}, \tag{39}$$

where ρ and τ are the amplitude coefficients of reflection and transmission of the beam splitter, respectively; in our case, they are $1/\sqrt{2}$. The transformation of the Fock states $|n_1\rangle$ and $|n_2\rangle$ at the inputs is described by the action of the beam splitter operator [38] as

$$\begin{aligned} \hat{B}|n_1, n_2\rangle &= \frac{1}{\sqrt{n_1 n_2}} \sum_{k_1,k_2}^{n_1,n_2} C_{k_1}^{n_1} C_{k_2}^{n_2} B_{11}^{k_1} B_{12}^{k_2} B_{21}^{n_1-k_1} B_{22}^{n_2-k_2} \\ &\times \sqrt{(k_1+k_2)!(n_1+n_2-k_1-k_2)!}\,|k_1+k_2, \\ &\quad n_1+n_2-k_1-k_2\rangle. \end{aligned} \tag{40}$$

For the state $|1{,}1\rangle$ at the input, there are two terms with states $|1{,}1\rangle$ at the output, while the coefficients preceding them are the same but have the opposite signs: $\tau^2|1{,}1\rangle - \rho^2|1{,}1\rangle$. This means that there will be no such state at the outputs.

How this result can be interpreted? It would seem, according to [38, p. 79], that it can be regarded as a manifestation of particle–wave dualism. In fact, on the one hand, photons behave like particles, showing single and double photocounts, and on the other hand, they behave as if they interfere at the beam splitter as waves with a certain phase difference. Which phase difference? Obviously, 0 or π, so that there are always be 2 or 0 photons at the outputs of the beam splitter. Thus, in fact, it is assumed that a certain phase difference exists in the photons that are mixed at the beam splitter, otherwise, there will be no effect of suppressing the correlation of photocounts. The presence of this phase difference is precisely the hidden parameter, which completely predetermines the outcome of the experiment; that is, we are dealing with the theory of hidden parameters in an explicit form. Let us see what consequences this interpretation has.

State $|1,1\rangle$ at the beam-splitter input can be obtained by parametric scattering. However, when the signal and idler beams of the parametric process are mixed at the beam splitter, the preparation of compressed states of light occurs, as characterized by the suppression of quantum fluctuations of one of the quadrature components of the field to the detriment of the other. Is the preparation compatible with the assumption that there must always be a phase difference of 0 or π between the signal and idler beams that follows from the above interpretation of the result of the experiment described in [28]?

We introduce the annihilation operators for the photons of the signal and idler beams, that is, $\hat{a}$ and $\hat{b}$. They are described by the Bogolyubov transformation of the operators of the seed vacuum modes $\hat{a}_0$ and $\hat{b}_0$:

$$\hat{a} = \mu\hat{a}_0 + \nu\hat{b}_0^+, \quad \hat{b} = \mu\hat{b}_0 + \nu\hat{a}_0^+. \tag{41}$$

In one of the channels, we introduce a phase delay θ. It is clear that it will not affect the effect of the suppression of the mutual correlation. This follows easily from the above consideration in the Heisenberg representation. However, how does the phase delay affect the preparation of compressed states? The mode annihilation operator for one of the beam splitter outputs is again denoted by $\hat{c} = (\hat{a} + \hat{b}e^{i\theta})/\sqrt{2}$. Accordingly, the quadrature component is $\hat{X} = (\hat{c} + \hat{c}^+)/2$. Let us find its variance

$$\begin{aligned} {}_a\langle 0|_b\langle 0|\hat{X}^2|0\rangle_b|0\rangle_a &= (2 + \mu\nu e^{i\theta} + \mu^*\nu^* e^{-i\theta})/8 \\ &= (1 + \mu\nu\cos\theta)/4 \end{aligned} \tag{42}$$

with real μ and ν. Here, the averaging is performed over the vacuum states of the initial seed modes and we used $|\mu|^2 + |\nu|^2 = 1$, which follows from the commutation relationships $[\hat{a}, \hat{a}^+] = [\hat{b}, \hat{b}^+] = 1$.

Thus, we have obtained that the compression effect essentially depends on the phase θ. This is understandable, because at one output of the beam splitter, light occurs in a compressed state, and on the other, on the contrary, it occurs with an increased dispersion of the quadrature, which corresponds to a phase change by π. Is this consistent with the assumption that the phase difference of the signal and idler beams oscillates with probability 1/2, taking the values of 0 or π? Obviously, it is not consistent, since in this case the compression effect would completely disappear upon averaging. This was found experimentally in [40], etc. Thus, the basic premise about the presence of a certain phase difference of photons leads to a logical contradiction, indicating its inadequacy.

Despite the absolute clarity and transparency of the result, its interpretation can be carried out from opposite mutually exclusive views. From the point of view of the Copenhagen interpretation, there really are no a priori specific values of the phase difference (its sine and cosine). The nonlocal theory of hidden parameters can cope well with the situation from its own positions. Indeed, if there is an instant nonlocal connection between all the objects that participate in the experiment and between the measurement results, then the phase difference of the photons of the entangled pair can have quite specific values, and those that correspond to the result of the experiment. To show the inadequacy of such an interpretation, we will modernize the experimental circuit.

Let us introduce a mode switch (Fig. 6, right) into the experiment circuit, which changes the direct detection of the right detector to balanced homodyne detection. The latter records the fluctuations of the quadrature component of the field (see, for example, [29] and citations therein) and if their level is below the vacuum state we can report the preparation of the compressed state. In the first phase of the experiment, along with the observation of the effect of suppression of the mutual correlation of photons, we record the rate of photocounts of each of the photodetectors. We then switch the circuit to the second mode, where the compressed state is observed. If, in this case, the phase difference between photons has changed from jumps of 0 or π to a constant phase difference, which is absolutely necessary for the preparation of the compressed state, then the photocounts in the left detector should change, because for a fixed phase of 0 or π either the photocounts are not observed at all or their number is twice as large. It is unlikely that this will happen. It is quite clear that the rate of photocounts will not change, because changing the conditions for recording radiation emitted from one beam-splitter channel cannot affect the results of recording in another. This simply follows from the principle of causality and elementary quantum calculations. Moreover, if it were possible, a system for instantaneous information transfer from one experimenter (with a mode switch) to another (with a left detector) would be designed immediately. Thus, *no specific phase difference* between photons can describe the result of the observations according to the circuit in Fig. 6. This only means that it simply does not exist. This, in turn, does not fit into the framework of *any theory* of hidden parameters, including nonlocal, precisely due to the mutually exclusive nature of the observed effects for a particular phase difference. The absence of a certain value of the measured quantity a priori means that this quantity is in the state of quantum superposition of all possible values. However, no theory of hidden parameters recognizes this fact, that is, quantum superposition. They are focused on any alternative in trying to explain the results of quantum experiments without involving the phenomenon of quantum superposition.

The phase difference of the correlated photon pair does not really have a definite value, but is in a superposition of all its possible values from 0 to 2π. How, then, should we interpret the effect of suppressing the mutual correlation of photocounts, if neither a certain phase of single photons nor their phase difference exist? Apparently, the issue is that, according to the Feimnan interpretation of the quantum theory [41], the alternative trajectories of photons interfere rather than the photons themselves. In fact, how is the $|1,1\rangle$ state formed at the beam splitter output? There are two possibilities: either both photons pass the beam splitter, or both are reflected. However, in the latter case, because one of the photons is reflected from a denser medium, it acquires a phase foray π (see also [42]). The phase foray operator of $\hat{U}_\theta = e^{-i\theta\hat{n}}$ transfers state $|1,1\rangle$ to state $-|1,1\rangle$. Thus, both possible alternative trajectories interfere destructively, thus suppressing mutual correlations. This simple and intuitive approach allows us to solve even more complex problems associated with the conversion of the Fock states by a beam splitter without using the complicated and cumbersome beam splitter operator (40).

The result obtained here is also important because the absence of definite values of the measured quantities before the moment of measurement is the fundamental conclusion of the quantum theory in the Copenhagen interpretation. The experimental evidence of this fact that is known so far can be challenged by involving allegedly unknown types of nonlocal interactions that are not limited to any area of the light cone and, correspondingly, the speed of light. These are, first, the interpretation of Bohm [43] and the following various types of nonlocal theories of hidden parameters (see, for example, [20–23, 44]), which can explain in a purely formal language both the violation of Bell's inequalities and numerous quantum paradoxes. As an example, the interference of single photons at a two-slit Young screen is interpreted as nonlocal "knowledge" of a photon passing through one slit about the existence of the other. In the experiment described here, there are good reasons for refuting such statements. No nonlocal "knowledge" of the photon about its future can explain the invariability of the rate of photocounts in the left detector. Thus, the absence of a definite value of the phase difference of single photons can in no way be challenged by any hypothesis of nonlocal interaction that does not reach the point of total absurdity. This significantly narrows the range of possible interpretations of the quantum theory, of course, not reducing them only to the Copenhagen interpretation. An adequate explanation can also be given by the relational paradigm, for example, [45, 46] and citations therein.

CONCLUSIONS

Experiments to observe the interference of a single photon show that the idea of a photon as a particle flying in space is at least naive. A quantized electromagnetic field in *some manifestations* (interaction with other objects, absorption, radiation, or scattering) behaves like particles—photons, while in other manifestations (propagation, interference, etc.) it acts as a field.

Surely, it would be unreasonable to doubt the physical reality of photons. We would like to emphasize once again the strange behavior of these quantum objects and, apparently, the impossibility of their representation in a visual model, that is, an exhaustive explanation of the nature of their behavior in the form of a *model* physical reality. Contrary to the existing prejudice, an electric field cannot be an analog of a wave function, as the former is a physical quantity that can be measured directly, while the latter is an information characteristic, the identification of which requires many experimental tests and quantum tomography [13, 38]. This difference is particularly pronounced in a single-photon standing wave: the electric component of the field at the nodes is zero, but does this mean that there is no photon there? By no means. After all, there is a magnetic component in the nodes, the presence of which can be detected by relativistic electrons that penetrate the nodes. Thus, the spatial distribution of a photon in a standing wave differs radically from the spatial distribution of the electric field.

Let us also emphasize that in quantum measurements, the absence of definite values of the measured quantities can a priori be considered an experimentally proven fact.

REFERENCES

1. G. I. Taylor, Proc. Cambridge Philos. Soc. **15**, 114 (1909).
2. O. Frish, Usp. Fiz. Nauk **90**, 379 (1966).
3. A. V. Belinskii, Phys.-Usp. **46**, 877 (2003).
4. A. V. Belinsky and D. N. Klyshko, Laser Phys. **6**, 1082 (1996).
5. J. S. Bell, Physics **1**, 195 (1964).
6. A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
7. B. Hensen, H. Bernien, A. E. Dréau, et al., Nature **526**, 682 (2015). doi doi 10.1038/nature15759
8. M. Giustina, M. A. M. Versteegh, S. Wengerowsky, et al., Phys. Rev. Lett. **115**, 250401 (2015).
9. A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981); A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).

10. A. Aspect, J. Dalibar, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
11. A. Aspect, *The Naive View of an Experimentalist* (Springer, 2002).
12. A. V. Belinsky, Opt. Spectrosc. **96**, 665 (2004).
13. A. V. Belinsky, *Quantum Measurements* (BINOM, Moscow, 2008).
14. A. V. Belinskii and D. N. Klyshko, Phys.-Usp. **36**, 653 (1993).
15. N. V. Evdokimov, D. N. Klyshko, V. P. Komolov, and V. A. Yarochkin, Phys.-Usp. **39**, 83 (1996).
16. B. Hessmo, P. Usachev, H. Heydari, and G. Björk, Phys. Rev. Lett. **92**, 180401 (2004).
17. S. A. Babichev, J. Appel, and A. I. Lvovsky, Phys. Rev. Lett. **92**, 193601 (2004).
18. M. Fuwa, S. Takeda, M. Zwierz, et al., Nat. Commun. **6**, 6665 (2015).
19. A. V. Belinsky and A. K. Zhukovskiy, Moscow Univ. Phys. Bull. **71**, 253 (2016). doi 10.3103/S0027134916030024
20. A. J. Leggett, Found. Phys. **33**, 1469 (2003).
21. M. Aspelmeyer and A. Zeilinger, Phys. World, No. 6, 22 (2008).
22. G. C. Knee, K. Kakuyanagi, M.-C. Yeh, et al., arXiv:1601.03728 [quant-ph].
23. S. Groblacher, T. Paterek, R. Kaltenbaek, et al., Nature **446**, 871 (2007).
24. C. Branciard, A. Ling, N. Gisin, et al., Phys. Rev. Lett. **99**, 210407 (2007).
25. V. Jacques, E. Wu, F. Grosshans, et al., Appl. Phys. Lett. **93**, 203307 (2008).
26. T. Paterek, A. Fedrizzi, S. Groblacher, et al., Phys. Rev. Lett. **99**, 210406 (2007).
27. A. Suarez, Nature **446**, 871 (2007).
28. C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).
29. S. A. Akhmanov, N. N. Akhmediev, A. V. Belinsky, et al., *New Physical Principles of Data Processing* (Nauka, Moscow, 1990), Chap. 3.
30. A. V. Belinsky and V. B. Lapshin, Moscow Univ. Phys. Bull. **71**, 258 (2016). doi 10.3103/S0027134916030036
31. D. N. Klyshko, Quantum Electron. **4**, 1056 (1977).
32. C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).
33. G. I. Taylor, Proc. Cambridge Philos. Soc. **15**, 114 (1909).
34. H. Hubel, D. R. Hamel, A. Fedrizzi, et al., Nature **466**, 601 (2010).
35. A. V. Burlakov, M. V. Chekhova, D. N. Klyshko, et al., Phys. Rev. A **56**, 3214 (1997).
36. D. N. Klyshko, Phys.-Usp. **37**, 1097 (1994).
37. A. N. Matveev, *Atomic Physics* (Moscow, 1989).
38. U. Leonhardt, *Measuring the Quantum State of Light* (Cambridge Univ. Press, 1997), p. 79.
39. D. N. Klyshko, *Photons and Nonlinear Optics* (Nauka, Moscow, 1980; CRC Press, 1988).
40. R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
41. R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw–Hill, 1965).
42. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge Univ. Press, 1995), Chap. 12.12.2.
43. D. Bohm, Phys. Rev. **85**, 166 (1952).
44. Yu. S. Efremov, *Quantum Mechanics* (Direkt Media, Moscow, 2015).
45. A. V. Belinsky, Laser Phys. **12**, 939 (2002).
46. A. V. Belinsky and Yu. S. Vladimirov, Prostranstvo, Vremya Fundam. Vzaimodeistviya, No. 1, 32 (2016).

Translated by O. Zhukova