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THEORETICAL AND MATHEMATICAL PHYSICS (REVIEW)

Decay Channels of the Standard Higgs Boson[1]

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Abstract—In this paper, we review the results of studies on the decay channels of the standard Higgs boson: $H \to f + \bar{f}$, $H \to Z + f + \bar{f}$, $H \to W + f + \bar{f}'$, $H \to \gamma + \gamma$, $H \to \gamma + Z$ and $H \to g + g$. Here $f\bar{f}$ or $f\bar{f}'$ are the fundamental fermions pair (leptons, quarks). Within the framework of the Standard Model analytical expressions for the partial widths of the indicated decays were obtained and their dependence on the mass of the Higgs boson was studied.

Keywords: standard model, Higgs boson, left and right coupling constants, helicity, Weinberg parameter, decay width.

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INTRODUCTION

The standard model (SM), based on local gauge symmetry $SU_C(3) \times SU_L(2) \times U_Y(1)$, describes well the physics of strong and electroweak interactions between leptons and quarks [1–6]. The theory introduces a doublet of scalar fields $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$, a neutral component, which has nonzero vacuum-wise value. As a result of spontaneous symmetry breaking, a new Higgs boson particle appears due to quantum excitations of the scalar field, and due to interaction with this field, gauge bosons ($W^{\pm}$- and Z^0), charged leptons and quarks acquire mass. This mechanism of mass generation of fundamental particles is known as the mechanism of spontaneous symmetry breaking of the Brout–Englert–Higgs symmetry [7–10]. However, until recently, the Higgs boson was not experimentally discovered. The search program for the scalar Higgs boson was one of the main tasks at the Large Hadron Collider (LHC) in CERN. The discovery of the Higgs boson with the characteristics corresponding to the SM predictions was carried out by the ATLAS and CMS collaborations in 2012 [11, 12] (see also the reviews [13–15]). The missing elementary particle in the SM building was found, by the discovery of a Higgs boson with a mass of 125 GeV and this is the beginning of new studies to elucidate the nature of this particle. In this connection, the theoretical interest in the various channels of decay and the production of the Higgs boson has greatly increased. Various properties of the Higgs-boson have been studied in [2, 16–21].

In this paper we review our studies on the decay channels of the Higgs boson:

$$H \to f + \bar{f}, \tag{1}$$

$$H \to Z + f + \bar{f}, \tag{2}$$

$$H \to W + f + \bar{f}', \tag{3}$$

$$H \to \gamma + \gamma, \tag{4}$$

$$H \to \gamma + Z, \tag{5}$$

$$H \to g + g, \tag{6}$$

where $f\bar{f}$ or $f\bar{f}'$ are pairs of fundamental fermions (leptons or quarks).

We note that the various decay channels of the Higgs boson were earlier studied by a number of authors [22–32]; some preliminary results were obtained by us in [33, 34]. However, these papers do not take into account the polarization states of finite particles. Below, our analysis shows that studying the polarization characteristics of particles can provide valuable information about the nature of the Higgs boson.

1. THE DECAY OF THE HIGGS BOSON INTO LEPTONS (QUARKS)

Let us first consider the decay of the Higgs boson into a fermion-antifermion pair (1). This process is

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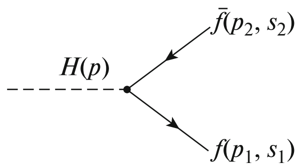


Fig. 1. The Feynman diagram of the decay $H \to f\bar{f}$.

described by the Feynman diagram and is shown in Fig. 1 (4-momentum and polarization vectors of particle are indicated in parentheses).

According to the SM, the spin of the Higgs boson is zero, and the P- and C-parities are positive: $J^{PC} = 0^{++}$. However, in addition to the scalar Higgs boson, the pseudoscalar A-boson is also discussed in the literature [2, 3]. In this regard, we consider a boson Φ that interacts with the fermion pair simultaneously contains the CP-even and CP-odd parts:

$$M(\Phi \to f\bar{f}) = \frac{m_f}{\eta}[\bar{u}(p_1,s_1)(a + b\gamma_5)\upsilon(p_2,s_2)]\Phi(p). \quad (7)$$

Here m_f is the fermion mass, $\eta = (\sqrt{2}G_F)^{-1/2} = 246$ GeV is the vacuum value of the Higgs-bosonic field, G_F is the Fermi constant of weak interactions, $\Phi(p)$ is the Φ-boson wave function normalized to unity, a and b are certain constants. For $a = 1$ and $b = 0$ (7) we obtain the decay amplitude of the standard Higgs boson while $a = 0$ and $b = 1$ give rise for the decay amplitude of the pseudoscalar A-boson.

The width of the decay of Φ-boson into the fermionic pair is proportional to the square of the matrix element:

$$\frac{d\Gamma}{d\Omega} = \frac{\beta_f}{64\pi^2 M_\Phi}\left|M(\Phi \to f\bar{f})\right|^2.$$

Calculating the square of the amplitude (7) with allowance for the arbitrary polarizations of the fermion and antifermion, in the rest frame of the Higgs boson will give us:

$$\begin{aligned}\frac{d\Gamma}{d\Omega} &= \frac{N_C\beta_f m_f^2}{128\pi^2}M_\Phi\sqrt{2}G_F \\ &\times\{|a|^2\beta_f^2[1 + (\boldsymbol{\xi}_1\boldsymbol{\xi}_2) - 2(\mathbf{n}\boldsymbol{\xi}_1)(\mathbf{n}\boldsymbol{\xi}_2)] \\ &+ |b|^2[1 - (\boldsymbol{\xi}_1\boldsymbol{\xi}_2)] + 2\,\mathrm{Re}(ab^*)\beta_f \\ &\times[(\mathbf{n}\boldsymbol{\xi}_1) - (\mathbf{n}\boldsymbol{\xi}_2)] + 2\,\mathrm{Im}(ab^*)\beta_f(\mathbf{n}[\boldsymbol{\xi}_1\boldsymbol{\xi}_2])\},\end{aligned} \quad (8)$$

where N_C is the color factor (for leptons $N_C = 1$ and $N_C = 3$ for quarks), $\mathbf{n}$ is the unit vector along the momentum of the fermion, $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are unit vectors directed along the spins of the fermion and antifermion in their rest systems and $\beta_f = \sqrt{1 - 4m_f^2/M_H^2}$ signifies the fermion velocity.

Suppose that the fermion-antifermion pair is polarized transversely ($\boldsymbol{\xi}_1 = \boldsymbol{\eta}_1$, $\boldsymbol{\xi}_2 = \boldsymbol{\eta}_2$, $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are the transverse components of the spin vectors of the fermions):

$$(\boldsymbol{\eta}_1\mathbf{n}) = (\boldsymbol{\eta}_2\mathbf{n}) = 0, \quad (\boldsymbol{\eta}_1\boldsymbol{\eta}_2) = \eta_1\eta_2\cos\varphi,$$
$$(\mathbf{n}[\boldsymbol{\eta}_1\boldsymbol{\eta}_2]) = \eta_1\eta_2\sin\varphi,$$

φ is the angle between the spin vectors $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$. In this case, the decay width $\Phi \to f + \bar{f}$ is:

$$\begin{aligned}\frac{d\Gamma(\varphi)}{d\Omega} &= \frac{N_C\beta_f m_f^2}{128\pi^2}\sqrt{2}G_F M_\Phi \\ &\times\{|a|^2\beta_f^2(1 + \eta_1\eta_2\cos\varphi) + |b|^2 \\ &\times(1 - \eta_1\eta_2\cos\varphi) + 2\,\mathrm{Im}(ab^*)\beta_f\eta_1\eta_2\sin\varphi\}.\end{aligned} \quad (9)$$

It follows that, if the transverse polarizations of the fermion and the antifermion are parallel ($\varphi = 0$), then for a complete transverse polarization of the fermions ($\eta_1 = \eta_2 = 1$), the decay of the Φ-boson can occur only due to the CP-even interaction:

$$\frac{d\Gamma(\varphi = 0)}{d\Omega} \sim \beta_f^2|a|^2.$$

The decay of the Φ-boson of the CP-odd interaction occurs only for anti-parallel transverse polarizations of the fermion-antifermion pair ($\varphi = \pi$):

$$\frac{d\Gamma(\varphi = \pi)}{d\Omega} \sim \beta_f|b|^2.$$

If the Φ-boson is a mixture of the CP-even and CP-odd states, then the asymmetry

$$A_1 = \frac{\frac{d\Gamma(\varphi = \pi/2)}{d\Omega} - \frac{d\Gamma(\varphi = -\pi/2)}{d\Omega}}{\frac{d\Gamma(\varphi = \pi/2)}{d\Omega} + \frac{d\Gamma(\varphi = -\pi/2)}{d\Omega}} = 2\eta_1\eta_2\frac{\mathrm{Im}(ab^*)}{|a|^2 + |b|^2} \quad (10)$$

will differ from zero and this asymmetry can reach values of the order of 1 (for complete transverse polarization of the fermions $\eta_1 = \eta_2 = 1$ and if they a and b are of the same order of magnitude).

For a pure CP-state, one of the coefficients a and b is equal to zero and another asymmetry

$$A_2 = \frac{\frac{d\Gamma(\varphi = 0)}{d\Omega} - \frac{d\Gamma(\varphi = \pi)}{d\Omega}}{\frac{d\Gamma(\varphi = 0)}{d\Omega} + \frac{d\Gamma(\varphi = \pi)}{d\Omega}} = \eta_1\eta_2\frac{|a|^2 - |b|^2}{|a|^2 + |b|^2} \quad (11)$$

will be either +1 or –1, depending on whether the Higgs boson is a CP-even or CP-odd state.

The aforementioned asymmetries A_1 and A_2 are more favorable to study in the decay channel of the Higgs boson $H \to \tau^- + \tau^+$. This is due to the fact that

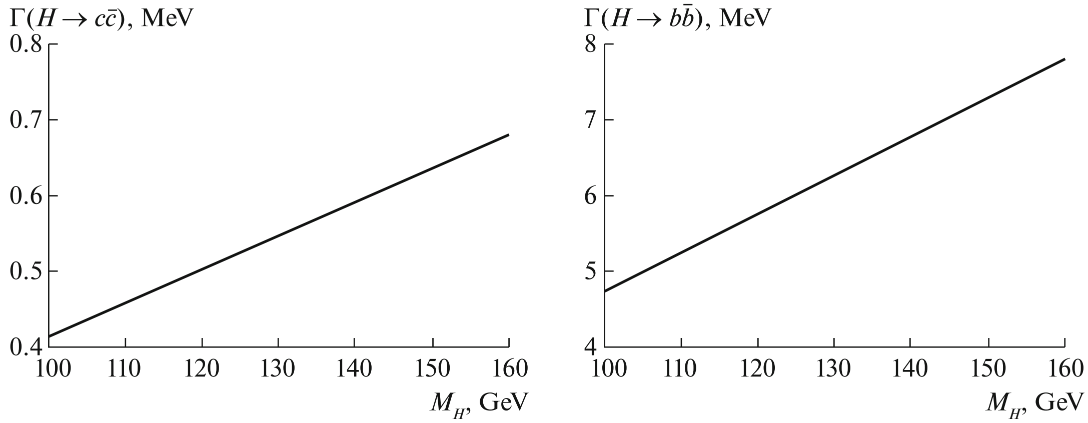


Fig. 2. Dependence of the width of decays $H \to c\bar{c}$ and $H \to b\bar{b}$ on the mass M_H.

the angular distribution of $\pi^-(\rho^-)$-mesons in the decay $\tau^- \to \pi^- + \nu_\tau(\rho^- + \nu_\tau)$ is very sensitive to spin τ-lepton, and thus the experiments will make it possible to measure the τ-lepton polarization.

Let us now consider the production of longitudinally polarized fermion-antifermion pair:

$$(\mathbf{n}\boldsymbol{\xi}_1) = \lambda_1,\ (\mathbf{n}\boldsymbol{\xi}_2) = -\lambda_2,\ (\boldsymbol{\xi}_1\boldsymbol{\xi}_2) = -\lambda_1\lambda_2,$$

where λ_1 and λ_2 is the helicity of the fermion and antifermion.

The width of the decay of the Φ-boson into a longitudinally polarized fermionic pair is as follow (the fermion masses are neglected in comparison with the mass of the Higgs boson):

$$\Gamma(\lambda_1, \lambda_2) = \frac{N_C}{32\pi}\beta_f M_\Phi m_f^2 \sqrt{2} G_F \\ \times [(|a|^2 + |b|^2)(1 + \lambda_1\lambda_2) + 2\,\mathrm{Re}(ab^*)(\lambda_1 + \lambda_2)]. \quad (12)$$

It follows from this expression that, in the decay $\Phi = f + \bar{f}$ the fermion and antifermion must have the same helicities ($\lambda_1 = \lambda_2 = \pm 1$). This is due to the law of conservation of the total angular momentum in decay $\Phi = f + \bar{f}$.

The difference in the decay $\Phi = f + \bar{f}$ probabilities for helicities $\lambda_1 = \lambda_2 = +1$ and $\lambda_1 = \lambda_2 = -1$ is the source of information about the interference of the CP-even and CP-odd amplitudes:

$$\Gamma(\lambda_1 = \lambda_2 = +1) - \Gamma(\lambda_1 = \lambda_2 = -1) \sim \mathrm{Re}(ab^*).$$

The sum of these probabilities is proportional to:

$$\Gamma(\lambda_1 = \lambda_2 = +1) + \Gamma(\lambda_1 = \lambda_2 = -1) \sim [|a|^2 + |b|^2].$$

The width of the decay of the boson, summed over the spin states of the fermion pair, is given by:

a) in the case of a scalar Higgs boson

$$\Gamma(H \to f\bar{f}) = \frac{G_F N_C}{4\sqrt{2}\pi} M_H m_f^2 \beta_f^3;$$

b) in the case of a pseudoscalar A-boson

$$\Gamma(A \to f\bar{f}) = \frac{G_F N_C}{4\sqrt{2}\pi} M_A m_f^2 \beta_f.$$

As can be seen, with the increase in the masses of the Higgs boson and the fermion, the decay probability $H \to f + \bar{f}$ increases. If $M_H \sim 125$ GeV, then the Higgs boson can decay into pair of $\tau^-\tau^+$-leptons, and pair of $c\bar{c}$-, $b\bar{b}$-quarks. Because of $M_H < 2m_t$, the Higgs boson can not decay into a pair of t-quarks. Because of the smallness of the masses of the electron, muon, u-, d-, and s-quarks, decays $H \to e^- + e^+$, $H \to \mu^- + \mu^+$, $H \to u + \bar{u}$, $H \to d + \bar{d}$, and $H \to s + \bar{s}$ are suppressed.

Figure 2 shows the dependence of the partial widths of the decays $H \to c + \bar{c}$ and $H \to b + \bar{b}$ on the mass of the Higgs boson at $m_c = 1.6$ GeV and $m_b = 4.8$ GeV. As was noted above, the width of the decays $H \to c + \bar{c}$ and $H \to b + \bar{b}$ increases with increasing the mass of Higgs boson.

2. THE DECAY OF THE HIGGS BOSON INTO $Z(W)$-BOSONS AND THE FERMION PAIR

If the mass of the Higgs boson is less than the sum of the masses of the gauge bosons ($M_H < 2M_Z$, $M_H < 2M_W$), then the decays $H \to Z + Z$ and $H \to W^+ + W^-$ are forbidden by the energy-momentum

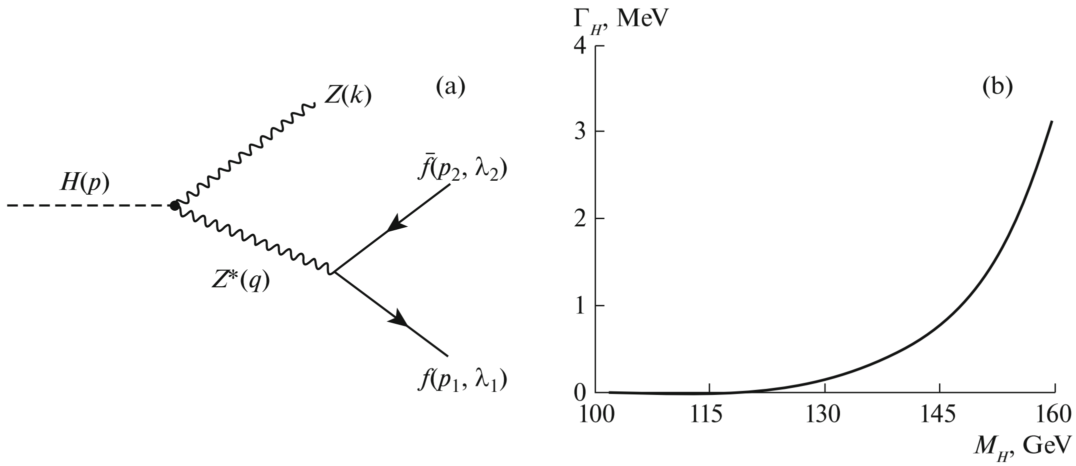


Fig. 3. (a) Feynman diagram of the reaction $H \to Zf\bar{f}$; (b) dependence of the width of the decay $\Gamma(H \to ZZ^*)$ on the mass M_H.

conservation laws. However, a Higgs boson can decay through channels

$$H \to Z + Z^* \to Z + f + \bar{f},$$
$$H \to W + W^* \to W + f + \bar{f}',$$

where Z^* and W^* are virtual bosons. First we consider the decay $H \to Z + Z^*$, the Feynman diagram of which is shown in Fig. 3a. The 4-momentum of the particles and the helicity of the fermions are shown in the figure. This diagram corresponds to the matrix element:

$$M(H \to Zf\bar{f}) = \frac{M_Z^2}{\eta} \times \frac{e}{\sin\theta_W \cos\theta_W} \frac{U_\mu^*(k)}{q^2 - M_Z^2 + iM_Z\Gamma_Z} \times \bar{u}(p_1,\lambda_1)\gamma_\mu[g_L(f)(1+\gamma_5) + g_R(f)(1-\gamma_5)]v(p_2,\lambda_2), \quad (13)$$

where

$$g_L(f) = I_3(f) - Q_f \sin^2\theta_W, \quad g_R(f) = -Q_f \sin^2\theta_W$$

are the left and right coupling constants of the fermion with the neutral Z-boson, $I_3(f)$ and Q_f are the third projection of the weak isospin and the electric charge of the fermion, θ_W is the Weinberg angle, $U_\mu^*(k)$—stands for the Z-boson polarization vector, M_Z and Γ_Z are the mass and the total width of the boson.

After squaring the amplitude of the process $H \to Z + f + \bar{f}$ we will have (the masses of the fermions are neglected):

$$\left|M(H \to Zf\bar{f})\right|^2 = \left(\frac{M_Z^2}{\eta}\right)^2 \times \frac{e^2 N_C}{x_W(1-x_W)} \frac{G_{\mu\nu}T_{\mu\nu}}{(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}, \quad (14)$$

where $x_W = \sin^2\theta_W$ is the Weinberg parameter,

$$G_{\mu\nu} = \sum_{pol.} U_\mu^*(k) U_\nu(k) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_Z^2}$$

is the tensor arising as a result of summation over the polarization states of the Z-boson and

$$T_{\mu\nu} = 2[g_L^2(f)(1-\lambda_1)(1+\lambda_2) + g_R^2(f)(1+\lambda_1)(1-\lambda_2)] \times [p_{1\mu}p_{2\mu} + p_{2\mu}p_{1\mu} - (p_1 \cdot p_2)g_{\mu\nu} - i\varepsilon_{\mu\nu\rho\sigma}p_{1\rho}p_{2\sigma}]$$

shows the tensor of the longitudinally polarized fermion-antiferional pair.

In the derivation of formula (14), we used the projection operators of longitudinally polarized particles [35]:

$$u(p_1,\lambda_1)\bar{u}(p_1,\lambda_1) = \frac{1}{2}(1 - \lambda_1\gamma_5)\hat{p}_1,$$

$$\upsilon(p_2,\lambda_2)\bar{\upsilon}(p_2,\lambda_2) = \frac{1}{2}(1 + \lambda_2\gamma_5)\hat{p}_2.$$

The product of tensors $G_{\mu\nu}$ and $T_{\mu\nu}$ is equal to:

$$G_{\mu\nu}T_{\mu\nu} = 2[g_L^2(f)(1-\lambda_1)(1+\lambda_2) + g_R^2(f)(1+\lambda_1)(1-\lambda_2)] \times\left[(p_1\cdot p_2) + \frac{2}{M_Z^2}(p_1\cdot k)(p_2\cdot k)\right]. \quad (15)$$

After introducing the invariant variables

$$s_1 = q^2 = (p_1+p_2)^2 = 2(p_1\cdot p_2),$$
$$s_2 = (k+p_2)^2 = M_Z^2 + 2(k\cdot p_2),$$
$$s_3 = (k+p_1)^2 = M_Z^2 + 2(k\cdot p_1),$$

which are related by

$$s_1 + s_2 + s_3 = M_H^2 + M_Z^2.$$

Choosing the independent variables s_1 and s_2, we have:

$$G_{\mu\nu}T_{\mu\nu} = [2g_L^2(f)(1-\lambda_1)(1+\lambda_2) + g_R^2(f)(1+\lambda_1)(1-\lambda_2)] \times\left[2s_1 + s_2 - M_H^2 + \frac{s_2}{M_Z^2}(M_H^2 - s_1 - s_2)\right]. \quad (16)$$

The invariant phase volume is given by

$$\int d\phi = \frac{1}{(2\pi)^5}\int\frac{d\mathbf{k}}{2E}\frac{d\mathbf{p}_1}{2E_1}\frac{d\mathbf{p}_2}{2E_2} \times\delta(p-k-p_1-p_2) = \frac{1}{128\pi^3 M_H^2}\iint ds_1 ds_2.$$

The boundaries of integration of the variables s_1 and s_2 are

$$0 \le s_1 \le M_H^2 + M_Z^2 - s_2 - \frac{M_H^2 M_Z^2}{s_2};$$
$$M_Z^2 \le s_2 \le M_H^2.$$

After integrating over the variables s_1 and s_2, for the width of the decay of the Higgs boson over the channel $H \to Z + f + \bar{f}$, we obtain:

$$\Gamma(H\to Zf\bar{f}) = \frac{\alpha N_C}{384\pi^2}\left(\frac{M_Z}{\eta}\right)^2 M_H\frac{R(x)}{x_W(1-x_W)} \times[g_L^2(f)(1-\lambda_1)(1+\lambda_2) + g_R^2(f)(1+\lambda_1)(1-\lambda_2)], \quad (17)$$

where the notation $x = (M_Z/M_H)^2$ is introduced and

$$R(x) = \frac{3(20x^2-8x+1)}{\sqrt{4x-1}}\arccos\left(\frac{3x-1}{2x\sqrt{x}}\right) - \frac{3}{2}(4x^2-6x+1)\ln x - \frac{1-x}{2x}(47x^2-13x+2). \quad (18)$$

From the formula for the decay width (17) it follows that the fermion and antifermion must have opposite helicities: $\lambda_1 = -\lambda_2 = \pm 1$. Hence, if the fermion is polarized to the left ($\lambda_1 = -1$), then the antifermion polarizes to the right ($\lambda_2 = +1$) and vice versa, if the fermion has right helicity ($\lambda_1 = +1$), then the antifermion will have left helicity ($\lambda_2 = -1$).

The width of the decay of the Higgs boson, summed over the helicity states of the fermion-antifermion pair, is given by the expression [34]:

$$\Gamma(H\to Zf\bar{f}) = \frac{\alpha N_C}{96\pi^2} \times\left(\frac{M_Z}{\eta}\right)^2 M_H\frac{g_L^2(f)+g_R^2(f)}{x_W(1-x_W)}R(x). \quad (19)$$

We note that the fundamental fermions are $f = \nu_e$, ν_μ, ν_τ, e^-, μ^-, τ^-, u, d, s, c, b (as noted above, pair of t-quarks can not occur during the decay of the Higgs boson). To find the total width of the decay of the Higgs boson over all possible $H\to Zf\bar{f}$ channels, it is necessary to find the sum $\sum_f N_C[g_L^2(f)+g_R^2(f)]$ over all fermions:

$$\sum_f N_C[g_L^2(f)+g_R^2(f)] = 3\left(\frac{7}{4} - \frac{10}{3}x_W + \frac{40}{9}x_W^2\right).$$

Then, for the width of the decay of the Higgs boson over all possible channels, we have, by the scheme $H\to Z + Z^*$,

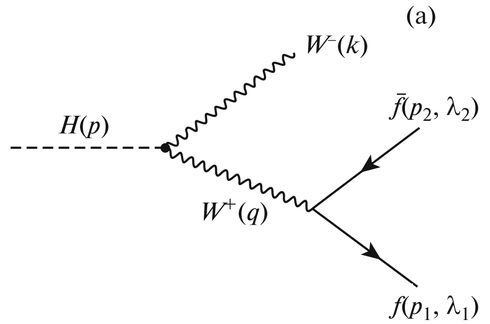
$$\Gamma(H\to ZZ^*) = \frac{\alpha}{32\pi^2}\frac{M_H}{x_W(1-x_W)} \times\left(\frac{M_Z}{\eta}\right)^2\left(\frac{7}{4} - \frac{10}{3}x_W + \frac{40}{9}x_W^2\right)R(x). \quad (20)$$

Figure 3b shows the dependence of the width of the decay of the Higgs boson on the mass M_H in the process $H\to Z + Z^*$ at the Weinberg parameter $x_W = 0.232$ and the mass of the Z-boson $M_Z = 91.1875$ GeV.

Now we consider the decay of the Higgs boson through the channel $H\to W^-W^{+*}\to W^-f\bar{f}'$, where $f = \nu_e, \nu_\mu, \nu_\tau, u, c$, and $\bar{f}' = e^+, \mu^+, \tau^+, \bar{d}, \bar{s}, \bar{b}$. The diagram of this process is shown in Fig. 4a.

We write the matrix element corresponding to this diagram:

$$M(H\to W^-f\bar{f}') = \frac{2M_W^2}{\eta} \times\frac{U_\mu^*(k)}{q^2 - M_W^2 + i\Gamma_W M_W}\frac{e}{2\sqrt{2}\sin\theta_w}U_{ff'} \times[\bar{u}(p_1,\lambda_1)\gamma_\mu(1+\gamma_5)v(p_2,\lambda_2)], \quad (21)$$



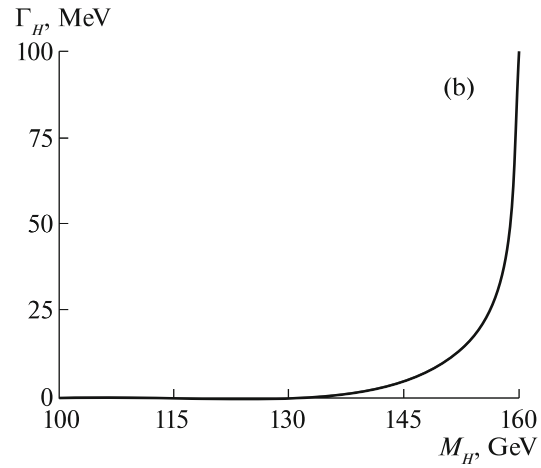


Fig. 4. (a) The Feynman diagram of the reaction $H \to W^- f\bar{f}'$; (b) dependence of the width of the decay $\Gamma(H \to WW^*)$ on the mass M_H.

when we have lepton pairs $\nu_e e^+$, $\nu_\mu \mu^+$, and $\nu_\tau \tau^+$ in the production then $U_{ff'} = 1$, but for the case of the creation of a pair of quarks $U_{ff'} = U_{qq'}$ will be the elements of the unitary Kobayashi-Maskawa matrix, and M_W and Γ_W are the mass and total width of the W-boson.

The square of the matrix element is defined in this form (the summation is carried out over the polarizations of the pair $f\bar{f}'$):

$$\sum |M(H \to W^- f\bar{f}')|^2 = \frac{e^2}{2x_W} \times \left(\frac{M_W^2}{\eta}\right)^2 |U_{ff'}|^2 \frac{G_{\mu\nu}T'_{\mu\nu}}{(q^2 - M_W^2)^2 + M_W^2\Gamma_W^2}, \tag{22}$$

where $G_{\mu\nu}$ and $T'_{\mu\nu}$ are the tensors of the W^--boson and the fermion pair. Their product is expressed by the formula

$$G_{\mu\nu}T'_{\mu\nu} = 4[2s_1 + s_2 - M_H^2] + \frac{s_2}{M_W^2}[M_H^2 - s_1 - s_2]. \tag{23}$$

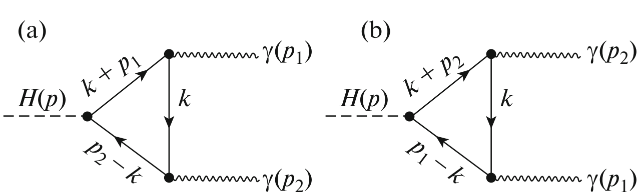


Fig. 5. Feynman diagrams of fermion loop for decay $H \to \gamma\gamma$.

There is also a decay channel for the Higgs boson according to the scheme $H \to W^+W^{-*} \to W^+ f\bar{f}'$. The square of the amplitude of this decay is also determined by the expression (22). Therefore, the total probability of all possible decays according to the scheme $H \to W + W^*$ is (for $\Gamma_W \to 0$) [20]:

$$\Gamma(H \to WW^*) = \frac{e^2}{2x_W M_H}\left(\frac{M_W^2}{\eta}\right)^2 \left[3 + N_C \sum_{q,q'} |U_{qq'}|^2\right] \times \int d\phi \frac{G_{\mu\nu}T'_{\mu\nu}}{(s_1 - M_W^2)^2} = \frac{3\alpha}{32\pi^2}\frac{M_H}{x_W}\left(\frac{M_W}{\eta}\right)^2 R(x), \tag{24}$$

where $R(x)$ is defined by the expression of (18), but in a given decay $x = (M_W/M_H)^2$.

Figure 4b shows the dependence of the decay width of the Higgs boson on the mass M_H in the process $H \to W + W^*$ under the Weinberg parameter $x_W = 0.232$ and $M_W = 80.425$ GeV.

3. THE DECAY OF THE HIGGS BOSON INTO PHOTONS

Since photons (gluons) are massless particles, they do not directly interact with Higgs bosons. The decay $H \to \gamma + \gamma$ goes through triangular diagrams with virtually charged particles. In Fig. 5 shows triangular fermion diagrams.

According to the gradient invariance, the matrix element of the decay $H \to \gamma + \gamma$ should have the form:

$$M(H \to \gamma\gamma) = A^\gamma e_\mu^{*(1)} e_\nu^{*(2)} [p_{2\mu} p_{1\nu} - (p_1 \cdot p_2) g_{\mu\nu}], \tag{25}$$

here $e_\mu^{*(1)}$ and $e_\nu^{*(2)}$ are the 4-vectors of polarization of the photons, p_1 and p_2 are their 4-momentum and

$A^{\gamma} = A_f^{\gamma} + A_W^{\gamma}$ characterize the contribution to the amplitude of the fermion and W-boson diagrams. Fermion loops are easily calculated on the basis of diagrams *a* and *b* in Figs. 5a and 5b. It is known that for fermions with higher masses the interaction with Higgs boson will be stronger so we can assume that the loop belongs to a heavy t-quark.

The matrix element corresponding to diagram in Fig. 5a is represented as

$$M_a(H \to \gamma\gamma) = N_C e^2 Q^2 \frac{m}{\eta} e_{\mu}^{*(1)} e_{\nu}^{*(2)} \times I_{\mu\nu}, \qquad (26)$$

where

$$I_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[\gamma_{\mu}(\hat{k} + \hat{p}_1 + m)(\hat{k} - \hat{p}_2 + m)\gamma_2(\hat{k} + m)]}{(k^2 - m^2)[(k + p_1)^2 - m^2][(k - p_2)^2 - m^2]}, \qquad (27)$$

$m = m_t$ and $Q = Q_t$ is the mass and charge of the heavy t-quark.

First, we compute the trace of the product of the matrices:

$$\mathrm{Tr}[\gamma_{\mu}(\hat{k} + \hat{p}_1 + m)(\hat{k} - \hat{p}_2 + m)\gamma_2(\hat{k} + m)] = 4mX_{\mu\nu},$$

where

$$X_{\mu\nu} = g_{\mu\nu}[m^2 - k^2 - (p_1 \cdot p_2)] + 4k_{\mu}k_{\nu} - 2k_{\mu}p_{2\nu} + 2k_{\nu}p_{1\mu} - p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}. \qquad (28)$$

Now, using the Feynman integration technique

$$\frac{1}{ABC} = \int_0^1 dx \int_0^1 dy \int_0^1 dz \delta(x + y + z - 1) \frac{2}{(Ax + By + Cz)^3},$$

we can calculate the integral $I_{\mu\nu}$, here

$$A = k^2 - m^2,$$
$$B = (k + p_1)^2 - m^2 = k^2 + 2(k \cdot p_1) + m^2,$$
$$C = (k - p_2)^2 - m^2 = k^2 - 2(k \cdot p_2) - m^2.$$

We simplify the expression:

$$Ax + By + Cz = (k^2 - m^2)x + [k^2 + 2(k \cdot p_1) - m^2]y + [k^2 - 2(k \cdot p_2) - m^2]z$$
$$= (k^2 - m^2) + 2(k \cdot p_1)y - 2(k \cdot p_2)z$$
$$= (k + p_1 y - p_2 z)^2 + 2(p_1 p_2)yz - m^2,$$

where the relations of $p_1^2 = p_2^2 = 0$ and $x + y + z = 1$ are used.

Introducing the notation $b^2 = m^2 - 2(p_1 \cdot p_2)yz$, we represent the integral in (27) in the form of

$$I_{\mu\nu} \equiv \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \int_0^1 dz \frac{8mX_{\mu\nu}}{[(k + p_1 y - p_2 z)^2 - b^2]^3}. \qquad (29)$$

Making the change of integration $k \to k - p_1 y + p_2 z$, we obtain (here the linear terms k are rejected and the following relations are taken into account $(e^{*(1)} \cdot p_1) = (e^{**(2)} \cdot p_2) = 0$):

$$I_{\mu\nu} \equiv \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \int_0^1 dz \frac{8mX'_{\mu\nu}}{(k^2 - b^2)^3}, \qquad (30)$$

where

$$X'_{\mu\nu} \equiv 4k_{\mu}k_{\nu} - k^2 g_{\mu\nu} + p_{2\mu}p_{1\nu}(1 - 4yz) + g_{\mu\nu}[m^2 - (p_1 \cdot p_2)(1 - 2yz)]. \qquad (31)$$

After integrating $I_{\mu\nu}$ first with respect to k, and then with respect to y and z, we will have the formula

$$I_{\mu\nu} = \frac{i}{4\pi^2 m}[p_{2\mu}p_{1\nu} - (p_1 \cdot p_2)g_{\mu\nu}]I_0, \qquad (32)$$

where

$$I_0 = \frac{1}{2\tau}\left[1 + \left(1 - \frac{1}{\tau}\right)\arcsin^2\sqrt{\tau}\right], \qquad (33)$$

$$\tau = \frac{M_H^2}{4m^2} < 1. \qquad (34)$$

Thus, for the matrix element of diagram in Fig. 5a we obtain the expression:

$$M_a(H \to \gamma\gamma) = -i\frac{N_C e^2 Q^2}{4\pi^2\eta} \times e_{\mu}^{*(1)} e_{\nu}^{*(2)}[p_{2\mu}p_{1\nu} - (p_1 \cdot p_2)g_{\mu\nu}]I_0. \qquad (35)$$

The same expression is obtained for the diagram in Fig. 5b. Therefore, the total amplitude is:

$$M(H \to \gamma\gamma) = 2M_a(H \to \gamma\gamma) = -i\frac{2N_C\alpha Q^2}{\pi\eta} e_{\mu}^{*(1)} e_{\nu}^{*(2)}[p_{2\mu}p_{1\nu} - (p_1 \cdot p_2)g_{\mu\nu}]I_0. \qquad (36)$$

Comparing expressions (26) and (36), for the contribution of the fermionic loop to the decay amplitude $H \to \gamma + \gamma$ we have

$$A_f^{\gamma} = -i\frac{g\alpha}{\pi M_W} N_C Q_f^2 I_0, \qquad (37)$$

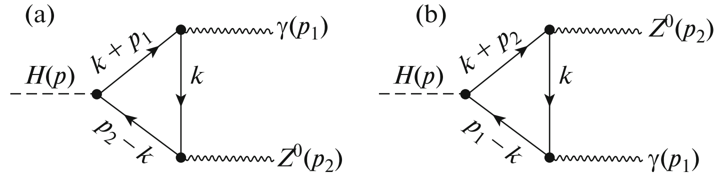


Fig. 6. Feynman diagrams of fermion loop decay $H \to \gamma Z$.

where g is the usual constant SM ($g^2/8M_W^2 = G_F/\sqrt{2}$).

For the probability of the decay of the Higgs boson into circularly polarized photons, the following expression is obtained:

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{32\sqrt{2}\pi^3} N_C^2 Q^4 I_0^2 (1 + l_1 \cdot l_2), \qquad (38)$$

where $l_1 = \pm 1$ and $l_2 = \pm 1$ characterize the circular polarization of photons. In the case of $l_1 = +1$ a photon has right, and in the case of $l_1 = -1$ left circular polarization.

From the probability decay formula (38) it follows that the γ-quantities must have either right ($l_1 = l_2 = +1$) or left ($l_1 = l_2 = -1$) circular polarizations. A state in which one of the γ-quantities has left and the other right circular polarization ($l_1 = -l_2 = \pm 1$) is forbidden by the conservation of the total angular momentum.

After summing over the circular polarizations of γ-quanta, the following expression was obtained for the decay $H \to \gamma + \gamma$ probability [33]:

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{8\sqrt{2}\pi^3} N_C^2 Q^4 I_0^2. \qquad (39)$$

We note that the diagrams also contribute to the decay $H \to \gamma\gamma$ with W-bosons loops, but these diagrams were not investigated by us. The calculation of such diagrams is presented in [2, 29–32].

4. THE DECAY OF A HIGGS BOSON INTO A PHOTON AND A Z-BOSON

Since the mass of the Higgs boson is larger than the mass of the Z-boson, decay by the scheme $H \to \gamma + Z$ is also possible, the Feynman diagrams of which are shown in Fig. 6.

As in the case of decay $H \to \gamma + \gamma$, the matrix element of the reaction $H \to \gamma + Z$ can be written as follows:

$$M(H \to \gamma Z) = A^Z e_\mu^* U_\nu^*(p_2)[p_{2\mu} p_{1\nu} - (p_1 \cdot p_2) g_{\mu\nu}], \qquad (40)$$

where e_μ^* and $U_\nu^*(p_2)$ is the 4-vector of the γ-quantum and Z-boson polarizations, $A^Z = A_f^Z + A_W^Z$ characterizes the contribution to the amplitude of fermion and W-boson loop diagrams.

To find the amplitude A_f^Z, we write the matrix element corresponding to diagram in Fig. 6a

$$M_a(H \to \gamma Z) = -ieQ \frac{N_C m}{\eta} \times \frac{e}{2\sin\theta_w \cos\theta_w} e_\mu^* U_\nu^*(p_2) I'_{\mu\nu}, \qquad (41)$$

where

$$I'_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[\gamma_\mu(\hat{k} + \hat{p}_1 + m)(\hat{k} - \hat{p}_2 + m)\Gamma_\nu(\hat{k} + m)]}{(k^2 - m^2)[(k + p_1)^2 - m^2][(k - p_2)^2 - m^2]}, \qquad (42)$$

$$\Gamma_\nu = \gamma_\nu[g_L(t)(1 + \gamma_5) + g_R(t)(1 - \gamma_5)],$$

and $g_L(t)$ and $g_R(t)$ are the left and right coupling constants of the t-quark with the Z-boson.

The trace of the product of Dirac matrices gives the expression

$$\mathrm{Tr}[\gamma_\mu(\hat{k} + \hat{p}_1 + m)(\hat{k} - \hat{p}_2 + m)\Gamma_\nu(\hat{k} + m)] = 4m[g_L(t) + g_R(t)]X_{\mu\nu} + 4im[g_L(t) - g_R(t)] \times \varepsilon_{\mu\nu\rho\sigma} p_{1\rho}(p_{2\sigma} - 2k_\sigma),$$

where the tensor $X_{\mu\nu}$ is represented by formula (28).

Again, we make a change of integration $k \to k - p_1 y + p_2 z$, and, using the relations $p_1^2 = 0$, $p_2^2 = M_Z^2$, $(e^* \cdot p_1) = 0$, $(U^*(p_2) \cdot p_2) = 0$, the integral (42) can be reduced to the form:

$$I'_{\mu\nu} \equiv \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \int_0^{1-y} dz \frac{8Y_{\mu\nu}}{(k^2 - c^2)^3}, \qquad (43)$$

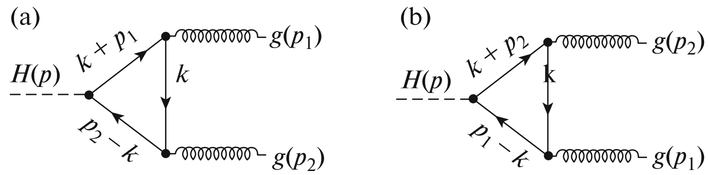


Fig. 7. Feynman diagrams for decay $H \to gg$.

where

$$Y_{\mu\nu} = g_V(t)\{4k_\mu k_\nu - k^2 g_{\mu\nu} + p_{2\mu}p_{1\nu}(1-4yz) + g_{\mu\nu}[m^2 - (p_1 \cdot p_2)(1-2yz) - M_Z^2 z^2]\} + ig_A(t)\varepsilon_{\mu\nu\rho\sigma}p_{1\rho}p_{2\sigma}(1-2z),$$

$$c^2 = m^2 + M_Z^2(z - z^2) - 2(p_1 \cdot p_2)yz,$$

$g_V(t) = g_L(f) + g_R(t)$ and $g_A(t) = g_L(t) - g_R(t)$ are the vector and axial coupling constants of the t-quark with the Z-boson.

An analogous integral is also obtained in the matrix element corresponding to diagram in Fig. 6b, however, the expression proportional to the axial coupling constant $g_A(t)$, reverses the sign. Consequently, the total contribution of both diagrams to the width of the decay $H \to \gamma + Z$ is proportional to the vectorial interaction constant of the t-quark $g_V(t)$:

$$A_f^Z = -\frac{ig\alpha N_C Q_f}{\pi M_W \cos\theta_W} g_V(f) I. \quad (44)$$

Here, I is the integral

$$I = \int_0^1 dy \int_0^{1-y} dz \cdot \frac{1-4yz}{1-4(\tau-\lambda)yz - 4\lambda z(z-1)}, \quad (45)$$

that depends on following parameters

$$\tau = \frac{M_H^2}{4m^2} < 1, \quad \lambda = \frac{M_Z^2}{4m^2} < 1.$$

This integral is expressed by elementary functions:

$$I = \frac{1}{2(\tau-\lambda)} + \frac{1}{(\tau-\lambda)^2} \times\left\{\frac{1}{2}(1-\tau+\lambda)[f_1(\lambda) - f_1(\tau)] - \lambda[f_2(\lambda) - f_2(\tau)]\right\}, \quad (46)$$

where

$$f_1(\lambda) = \arcsin^2(\sqrt{\lambda}), \quad f_2(\lambda) = \sqrt{\frac{1-\lambda}{\lambda}}\arcsin(\sqrt{\lambda}). \quad (47)$$

The following expression was obtained for the decay width [33]:

$$\Gamma(H \to \gamma Z) = \frac{1}{32\pi}\left(\frac{g\alpha}{\pi M_W}\frac{N_C Q}{\cos\theta_W} g_V(t)\right)^2 \times M_H^3\left(1 - \frac{M_Z^2}{M_H^2}\right)^3 I^2. \quad (48)$$

The decay $H \to \gamma + Z$ is also contributed by W-bosons loop diagrams, but this contribution is not discussed here (see [2]).

5. THE DECAY OF THE HIGGS BOSON INTO TWO GLUONS

This process is also described by two Feynman diagrams, shown in Fig. 7.

First, after writing the matrix element corresponding to diagram a):

$$M_a(H \to gg) = -ig_s^2 \frac{m}{\eta}\varepsilon_{1\mu}^{*a}\varepsilon_{2\nu}^{*b}\mathrm{Tr}\left(\frac{\lambda_a}{2}\cdot\frac{\lambda_b}{2}\right) I_{\mu\nu}, \quad (49)$$

in which g_s is the quark-gluon interaction constant, $\varepsilon_{1\mu}^{*a}$ and $\varepsilon_{2\nu}^{*b}$ are the gluon polarization vectors, λ_a are the Gell–Mann matrices, and the integral $I_{\mu\nu}$ is given by expression (27).

The trace of these matrices is defined as

$$\mathrm{Tr}\left(\frac{\lambda_a}{2}\cdot\frac{\lambda_b}{2}\right) = \frac{1}{4}\mathrm{Tr}(\lambda_a\lambda_b) = \frac{1}{2}\delta_{ab}.$$

Further calculations were carried out as well as in the decay $H \to \gamma + \gamma$, and for the total reaction $H \to g + g$ the amplitude expression became

$$M_a(H \to gg) = -ig_s^2 \frac{m_q}{\eta}\varepsilon_{1\mu}^{*a}\varepsilon_{2\nu}^{*b}\delta_{ab} I_{\mu\nu}, \quad (50)$$

where the tensor $I_{\mu\nu}$ is given by (32).

Summing over the color and polarization states of gluons were carried according to the rules

$$\sum_{a,b}\delta_{ab}\delta_{ab} = \sum_{a,b}\delta_{aa} = 8,$$

$$\sum \varepsilon_{1\mu}^{*a}\varepsilon_{1\rho}^{a}\varepsilon_{1\nu}^{*b}\varepsilon_{1\sigma}^{b} = \delta_{\mu\rho}\delta_{\nu\sigma},$$

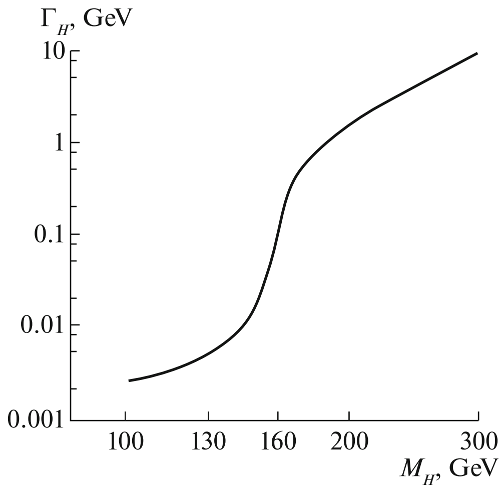


Fig. 8. Dependence of the total width of the decay of the Higgs boson on its mass.

and by squaring the amplitude of the decay $H \to g + g$:

$$\left|M(H \to gg)\right|^2 = 8g_s^4\left(\frac{m}{\eta}\right)^2 I_{\mu\nu}I_{\mu\nu}^* = 4\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{M_H^2}{\eta}\right)^2 I_0^2. \quad (51)$$

For the width of the decay of the Higgs boson according to the scheme $H \to g + g$, will give us the expression [33]

$$\Gamma(H \to gg) = \frac{1}{8\pi}\frac{M_H^3}{\eta^2}\left(\frac{\alpha_s}{\pi}\right)^2 I_0^2. \quad (52)$$

At a mass of the Higgs-boson $M_H = 125$ GeV, the partial width of the decay $H \to g + g$ is $\Gamma(H \to gg) \sim 0.2$ MeV and increases with increasing mass of the Higgs boson, for example, at a $M_H \approx 400$ GeV mass, the decay width is $\Gamma(H \to gg) \sim 10$ MeV.

CONCLUSION

Thus, we studied various decay channels of the standard Higgs boson: decays into lepton and quark pairs ($H \to \tau^-\tau^+$, $H \to c\bar{c}$, $H \to b\bar{b}$) decays into real and virtual gauge bosons ($H \to ZZ^*$, $H \to WW^*$), decays into photons ($H \to \gamma\gamma$), $H \to \gamma Z$ decaying and decays into gluons ($H \to gg$). We have obtained analytic expressions for the amplitudes and probabilities of all the indicated decays in the framework of the SM, Analyzing the obtained expressions show that in the mass spectrum of the Higgs boson, 100 GeV $\le M_H \le$ 130 GeV, the main mode of decay is $H \to b\bar{b}$. For example, at $M_H = 120$ GeV, the relative probabilities of various channel decays are

$$B(b\bar{b}) = \frac{\Gamma(H \to b\bar{b})}{\Gamma(H \to \text{all})} \approx 68\%, \quad B(\tau^-\tau^+) \approx 7\%,$$

$$B(c\bar{c}) \approx 3\%, \quad B(gg) \approx 7\%,$$

$$B(WW^*) = 13\%, \quad B(ZZ^*) \approx 2\%,$$

and other decay channels are very small.

With increasing the mass of the Higgs boson, the decay $H \to WW^*$ and $H \to ZZ^*$ probabilities will increase. For example, at M_H=300 GeV we have

$$B(WW^*) \approx 69\%, \quad B(ZZ^*) \approx 30\%.$$

Figure 8 depicts the dependence of the decay $\Gamma(H \to \text{all})$ probability on the mass of the Higgs boson. Due to decays $H \to WW^*$ and $H \to ZZ^*$, with increasing mass M_H, the total width of the decay will increase.

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