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Threshold Fields for Stimulated Brillouin Scattering in Spatially Limited Plasma

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Abstract—Brillouin scattering in an infinite medium is anisotropic, in this case the threshold of absolute instability is caused by attenuation of scattered waves. If the collision attenuation mechanism prevails, the minimum threshold value is observed during backward scattering. For a scattering region limited in the longitudinal direction (parallel to the direction of pumping wave propagation), the backward scattering threshold will be greater than for an infinite medium due to convective loss associated with energy removal by scattered waves. In this paper, the scattering of a wide wave beam in plasma is considered, whose dimension in the transverse direction to the pumping wave propagation substantially exceeds the dimension in the longitudinal direction. It was revealed that in this case, during angle scattering the instability threshold can be less than the threshold for backward scattering due to the increased time of radiation removal from the interaction region. This effect was not taken into account previously. In turn, the decrease of the threshold leads to increasing the radiation loss, which is important in plasma heating problems. The results can also be used for plasma diagnostics.

Keywords: stimulated Raman scattering, SRS, stimulated Brillouin scattering, SBS, ion-acoustic wave.

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INTRODUCTION

The interest in the problems of stimulated Raman scattering has been supported for a long time in connection with the problems of accelerating electrons by a laser beam [1], laser thermonuclear fusion [2], compression and gain of laser pulses [3], plasma diagnostics [4] and other problems. In the first works dedicated to the analysis of instabilities in an infinite medium ([5, 6], see also [7]), it was revealed that the scattering is anisotropic, the instability threshold is minimal in backward scattering, and its value is determined by the attenuation of scattered waves.

Scattering theory in limited plasma developed in three directions: numerical modeling [8, 9], rigorous analytical solution of problem [6, 10–12], and study of the scattering of short pulses, for which the propagation of a low-frequency wave during an impulse can be negligible [13–17]. The first direction gives interesting examples of the development of instability, however, due to its inconvenience does not make it possible to carry out a systematic research on a wide range of parameters. The significant mathematical simplification of the problem achieved in the latter case [13–17] made it possible to investigate a variety of problems of particle acceleration and compression of laser pulses. At the same time for the problems of plasma heating and plasma diagnostics in general this approximation cannot work. The complete problem of scattering in limited plasma [6, 7, 10–12] was analytically solved only for backward scattering (by angle π) for collisionless plasma. In this case, the instability threshold increases due to additional (as compared to infinite plasma) energy removal from the interaction region by the scattered waves.

This paper considers the scattering of the wide beam of a pumping wave on a limited plasma, the dimension of which in the direction across the pumping-wave propagation we will assume to be infinite, at an arbitrary value of the scattering angle of an electromagnetic wave. This problem can serve as a first approximation for calculating the cases when the lateral dimension of a plasma is much larger than the longitudinal dimension L. Since the runaway time of scattered radiation increases during deviation of the scattering angle from π, it can be expected that the threshold of absolute instability will decrease. At angles close to $\pi/2$ the removal time of radiation is large and the threshold will be determined by the

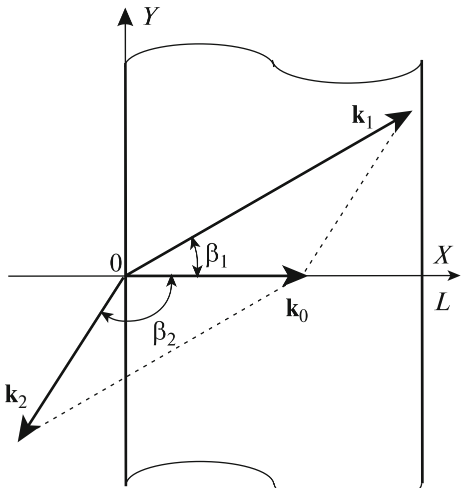

Fig. 1. The location of the wave vectors for backward scattering. $\mathbf{k}_0$ is the wave vector of pumping wave, $\mathbf{k}_1$ is the wave vector of sound wave, $\mathbf{k}_2$ is the wave vector of electromagnetic scattered wave. The plasma dimension along axis $0Y$ and the beam width of pumping wave is much more than longitudinal dimension L.

absorption of the scattered waves. The convective energy loss decreases in this case and the threshold approaches the threshold in an infinite plasma.

THE SOLUTION OF A SHORTENED SYSTEM OF EQUATIONS

The geometry of the interaction region in which the scattering is considered is shown in Fig. 1. A pumping wave with an amplitude of E_0, a frequency of ω_0, and a wave vector $\mathbf{k}_0$ propagates along the $0X$ axis and the interaction region is filled by homogeneous plasma with a density of n_0 and is limited in the direction of pumping-wave propagation by the coordinates $0 < X < L$. For the effects discussed in the paper to be observed, the dimensions of the beam and plasma along axis $0Y$ should be much more than L. Within the framework of the model problem, both these dimensions are assumed to be infinite. In the direction perpendicular to the $X0Y$ plane ($0Z$ axis) the beam dimension can be both smaller and larger than the plasma dimension; however, the minimum of these dimensions, as well as the longitudinal dimension L, should be much greater than the pumping-wave wavelength. From the point of view of the processes discussed in the paper the method of plasma creation is not essential.

We assume that a sound wave (with a frequency of Ω_1 and the wave vector of $\mathbf{k}_1$) propagates in a positive direction on the $0X$ and $0Y$ axes and a scattered electromagnetic wave (ω_2 and $\mathbf{k}_2$) propagates in the negative direction of the axes (Fig. 1). During the scattering the synchronism conditions are fulfilled

$$\Omega_1 + \omega_2 = \omega_0, \quad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_0. \tag{1}$$

As shown in [18, 19], since all three wave vectors $\mathbf{k}_0$, $\mathbf{k}_2$ and $\mathbf{k}_1$ lie in one plane, the scattering process can be described by a set of shortened equations for the waves with the amplitudes depending only on two spatial variables, which we denote X and Y. Without restriction of generality one can consider that the vectors of the electric field strength of the pumping wave and of the scattered electromagnetic wave are directed along the $0Z$ axis (Fig. 1) [18, 19] because for a different polarization the difference lies only in the reduction of the coefficients of nonlinear interaction of waves and leads to increasing the instability thresholds. Taking this effect into account is not difficult, but it leads to formulas that are more cumbersome. Accepting that the scattering angles of the soundwave and electromagnetic wave are equal to β_1 and β_2 (Fig. 1), we can write a shortened equation for waves [12, 18, 20, 21] in the form

$$\begin{aligned} &\frac{\partial b_1}{\partial T} + (\mathbf{v}_1\nabla)b_1 + \Gamma_1 b_1 = \tilde{M}_1 b_2, \\ &\frac{\partial b_2}{\partial T} + (\mathbf{v}_2\nabla)b_1 + \Gamma_2 b_2 = M_2 b_1. \end{aligned} \tag{2}$$

Here, $V_{1x} = V_1\cos\beta_1$, $V_{1y} = V_1\sin\beta_1$, $V_{2x} = V_2\cos\beta_2$, $V_{2y} = V_2\sin\beta_2$, T is time and X, Y are the coordinates. In the simplest case of Brillouin scattering $b_1 = \delta n_e^*/n_0$ is the amplitude of the sound wave, δn_e is the perturbation of the electron density, $b_2 = E_2$ is the amplitude of the scattered electromagnetic wave, $\mathbf{v}_1$, $\mathbf{v}_2$ are the group velocities of waves for Brillouin scattering $V_1 = V_S$, and $V_2 = c$, where V_S is sound wave speed and c is the speed of light. The nonlinear coupling coefficients of the waves equal $\tilde{M}_1 = ze^2|k_1|E_0/4mm_i\omega_0\omega_2V_S$, $\tilde{M}_1 = M_1|k_1|/k_0$, $M_2 = \omega_{Le}^2E_0/4\omega_0$. The remaining coefficients of equation are expressed by the plasma parameters $\Gamma_1 = \nu_{in}/2$, $\Gamma_2 = \nu_e\omega_{Le}^2/2\omega_2$ are the attenuation decrements of sound and electromagnetic waves; ν_{in}, ν_e are the rates of elastic collisions of ions and electrons; e, m are the electron charge and mass; z, m_i are the charge number and mass of the ions; and ω_{Le} is the Langmuir frequency of electrons. At infinity the Sommerfeld radiation conditions for scattered waves are set (Fig. 1), which for system of equations (2) is reduced to the form

$$b_1(x = 0) = 0, \quad b_2(x = L) = 0. \tag{3}$$

For waves amplitudes b_1 and b_2 the initial conditions $b_1 = b_{10}(X, Y)$, $b_2 = b_{20}(X, Y)$ are set. The total scattering field can be calculated as a superposition of waves scattered under different angles. Owing to the fact that the speed of sound waves is much less than the speed of light, we will assume that the wavenumbers of the scattered electromagnetic waves and pumping waves are equal; it follows from Eq. (1) that $k_1 = 2k_0\sin(\beta_2/2)$.

It is convenient to record system (2) in dimensionless variables. We introduce the new coordinates $x = X/L$, $y = Y/L$, time $t = cT/L$, and the mean free paths of sound wave $l_S = V_S/\Gamma_1$ and of scattered wave $l_t = c/\Gamma_2$ that represent the specific distances, at which the amplitude of free waves decreases by e times. When discussing the results, it is also convenient to use the parameters that characterize the intensity of the pumping wave $p_0 = 2\sqrt{M_1M_2/cV_S}$, $1_E = 2/p_0$ and the parameter of threshold exceeding $\lambda = p_0L/2 = L/l_E$. The effect of attenuation of scattered waves is estimated by the parameter $q_2 = L/l_S(1/\sin(\beta_2/2)) - l_S/l_t\cos(\beta_2)$, which we will call the dimensionless attenuation coefficient. Note that in our designations $\cos(\beta_2) < 0$. The minimum value of parameter q_2 equals $q_{20} = L/l_S + L/l_t$ and is observed when $\beta_2 = \pi$. After the introduction of new symbols we obtain

$$\left(\frac{c}{V_S}\frac{\partial}{\partial t} + \sin\left(\frac{\beta_2}{2}\right)\frac{\partial}{\partial x} + \cos\left(\frac{\beta_2}{2}\right)\frac{\partial}{\partial y} + \frac{L}{l_S}\right) = \xi\frac{L}{l_E}\sin\left(\frac{\beta_2}{2}\right)b_2,$$
$$\left(\frac{\partial}{\partial t} + \cos\beta_2\frac{\partial}{\partial x} - \sin\beta_2\frac{\partial}{\partial y} + \frac{L}{l_t}\right)b_2 = \xi^{-1}\frac{L}{l_E}b_1, \tag{4}$$

where $\xi = \sqrt{cM_1/V_SM_2}$. Due to the linearity of the problem and the homogeneity of the equations under consideration under the y coordinate, the dependence of the slow amplitudes on y coordinates can be taken into account by changing the y-components of the wave numbers of scattered waves under arbitrary initial conditions $b_1 = b_{10}(x, y)$, $b_2 = b_{20}(x, y)$. Therefore, without restriction of generality one can consider that the initial perturbations do not depend on y. The solutions for these initial conditions will satisfy the simplified variant of Eqs. (4) in which the relationship $\partial/\partial y = 0$ is fulfilled. The final equations with an accuracy of notation are similar to those discussed in [11, 12, 21–24]. Their difference is that they describe the evolution of instability for the arbitrary scattering angle of the electromagnetic wave β_2, and not only during backward scattering ($\beta_2 = \pi$). To solve these equations, first the Laplace transform by t is used

$$\begin{pmatrix} B_1(p,x) \\ B_2(p,x) \end{pmatrix} = \int_0^\infty \begin{pmatrix} b_1(t,x) \\ b_2(t,x) \end{pmatrix} \exp(-pt)dt,$$

the obtained equations are then considered as a linear problem of the eigenvalues for the argument of the Laplace transform p. The obtained dispersion equation

$$\Delta = \left(\frac{c}{V_S\sin(\beta_2/2)}p + \frac{L}{l_S\sin(\beta_2/2)} + i\chi_1\right)e^{i\chi_1L} - \left(\frac{c}{V_S\sin(\beta_2/2)}p + \frac{L}{l_S\sin(\beta_2/2)} + i\chi_2\right)e^{i\chi_2L} = 0. \tag{5}$$

where

$$\chi_{1,2} = \frac{i}{2}\left\{\left(\frac{c}{V_S\sin(\beta_2/2)} + \frac{1}{\cos(\beta_2)}\right)p + \left(\frac{L}{l_S\sin(\beta_2/2)} + \frac{L}{l_t\cos(\beta_2)}\right) \pm\sqrt{\left[\left(\frac{c}{V_S\sin(\beta_2/2)} - \frac{1}{\cos(\beta_2)}\right)p + \left(\frac{L}{l_S\sin(\beta_2/2)} - \frac{L}{l_t\cos(\beta_2)}\right)\right]^{2^*} - \frac{4}{\cos(\beta_2)}\left(\frac{L}{l_E}\right)^2}\right\}.$$

associates the eigenvalues of the problem (3), (4), which we will denote as p_n, and the dimensionless parameters c/V_S, L/l_S, L/l_t, and $\lambda = L/l_E$. It is known that the system (3)–(4) has a finite number of real eigenvalues and an enumerable set of complex eigenvalues [6, 11, 12, 24]. The positive values p_n that satisfy Eq. (5) are the dimensionless increments of the instability of the corresponding spatial modes. The solution of Eq. (5) is convenient to write by introducing the variable $\vartheta = \arccos\left(\left(\left(\frac{c}{V_S\sin(\beta_2/2)} - \frac{1}{\cos(\beta_2)}\right)p + \left(\frac{L}{l_S\sin(\beta_2/2)} - \frac{L}{l_t\cos(\beta_2)}\right)\right)/\left(\frac{2L}{l_E\sqrt{-\cos(\beta_2)}}\right)\right)$, which satisfies the equation (n is the number of the unstable mode)

$$\vartheta + \frac{\lambda}{\sqrt{-\cos(\beta_2)}}\sin\vartheta = \pi n. \tag{6}$$

The same Eq. (5) determines the threshold intensity of the pumping wave λ_n when $p_n = 0$ for the mode n as a function of $L/1_t$ and $L/1_E$. Along with the threshold field, the different modes are distinguished by the spatial distribution of the wave amplitudes: the mode with the number $n = 1$ has $(n - 1)$ sign inversions on the segment $0 < x < L$. The plots of eigenfunctions for various modes were presented, for example, in [11, 24]. After cumbersome transformations we

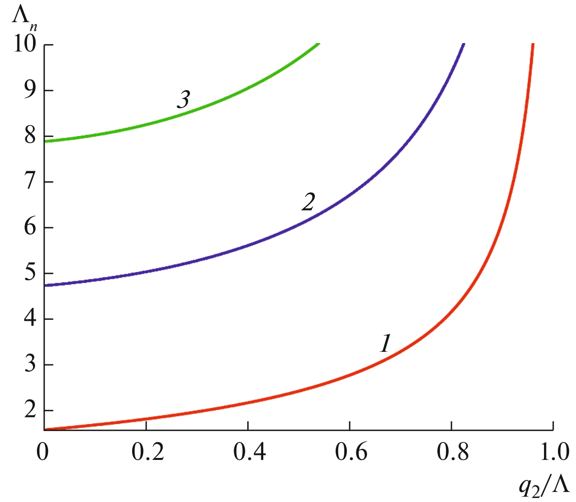

Fig. 2. The dependence of the instability threshold on the ratio of the dimensionless attenuation coefficient q_2 to the intensity of pumping wave Λ for 1, 2 and 3 modes (the figure at the curve is a mode number).

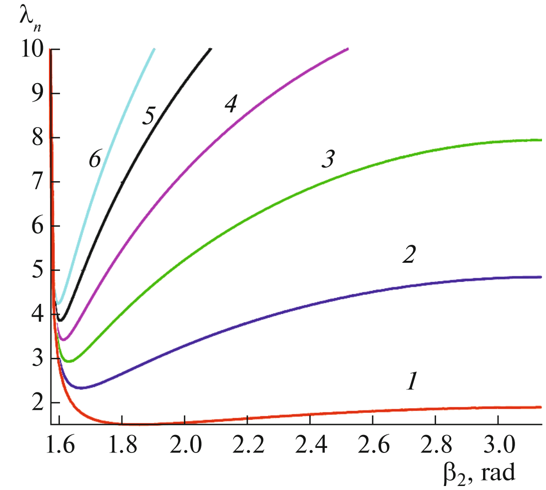

Fig. 3. The dependence of the instability threshold on the scattering angle β_2 (rad) for a low attenuation coefficient ($q_2/p_0 = 0.01$) (the figure at the curve is the mode number).

obtain that the dimensionless threshold of instability λ_n for mode n is determined by the expression

$$\lambda_n \frac{2}{\sqrt{-\cos(\beta_2)}} \cos\vartheta_n - \frac{L}{l_S \sin(\beta_2/2)}\left(1 - \frac{l_S}{l_t}\frac{\sin(\beta_2/2)}{\cos(\beta_2)}\right) = 0, \quad (7)$$

where ϑ_n satisfies Eq. (6). For calculating the threshold strength of the electric field it is convenient to represent the relationship λ with the parameters of the plasma and electric field strength of the pumping wave in the form of the product of several dimensionless factors $\lambda = \left(\frac{eE_0}{m\omega_0 V_{Te}}\right)\left(\frac{\omega_{Le}}{\omega_0}\right)(k_0 L)\sqrt{\frac{\omega_0}{\omega_1}}$, where $V_{Te} = (kT_e/m)$ is the electron thermal velocity. In the collisionless mode l_S and l_t tend to ∞, while the second item in (7) becomes zero. Collisions lead to increasing the threshold in accordance with (7). One can obtain the necessary condition of instability from (7) in a similar manner to [22]

$$\frac{q_2}{\Lambda} < 1, \quad (8)$$

where $\Lambda = \lambda/\sqrt{-\cos(\beta_2)}$. The dependence of the instability threshold $\Lambda_n = \lambda_n/\sqrt{-\cos(\beta_2)}$ on parameter q_2/Λ defined by Eqs. (7) is shown in Fig. 2. It is similar to the dependence obtained in [22] for convective instability during scattering in the passing direction. The instability thresholds for all of the waves increase with the growth of q_2/Λ and reach infinity when this parameter equals unity.

When increasing the deviation of the scattering angle β_2 from π the energy loss of scattered waves in the collisions increases. In contrast, the convective losses are at a maximum under strictly backward scattering $\beta_2 = \pi$ and sharply decrease with $\beta_2 \to \pi/2$. The dependences of the loss on the angle are characterized by the fact that during lateral scattering the losses associated with the attenuation of scattered waves always predominate and the convective losses are small. If the attenuation of waves is sufficiently small and when $\beta_2 = \pi$ the convective losses predominate; then, in the intermediate range of angles the minimum of the threshold field will be observed. As an example, Fig. 3 shows the angular dependence of the instability threshold under these conditions for different unstable modes. The scattering angle β_2 in Fig. 3 and subsequent figures is measured in radians.

When large losses are associated with the attenuation of scattered waves, the minimum of the dependence of the threshold of lower modes on the scattering angle will be absent (Fig. 4), although we see that it can be observed on the dependence of the threshold of higher modes for which the convective losses are higher than one for the first mode.

In most mediums the condition of $l_S/l_t \ll 1$ is fulfilled, i.e., the mean free path for the sound wave is much smaller than for the electromagnetic wave. The collisional absorption of the sound wave changes relatively small when changing the scattering angle. Therefore, the absorption of a transverse wave (the last item $l_S \sin(\beta_2/2)/l_t \cos(\beta_2)$ in Eq. (7)) substantially affects the increment of instability only for scattering angles close to $\pi/2$. Increasing the scattering angle leads to a rapid decrease in the influence of the

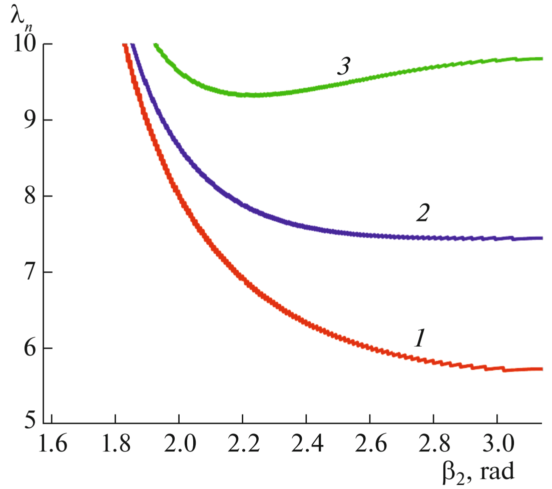

Fig. 4. The dependence of the instability threshold on the scattering angle β_2 with a relatively large attenuation coefficient ($q_{20}/\lambda = 0.1$). The mean free path of the sound wave l_S is much less than the mean free path of the electromagnetic wave l_t. The calculation was carried out at $l_S = 0.01 l_t$.

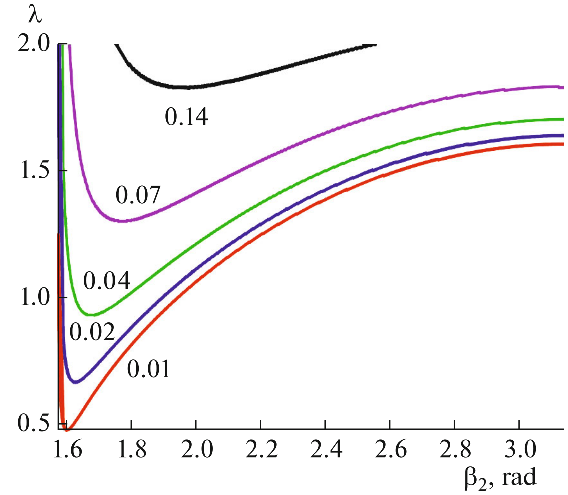

Fig. 5. The dependence of the instability threshold on the scattering angle β_2 with different values of the dimensionless attenuation coefficient q_{20} (the figure at the curve is parameter q_{20}, $l_S = 0.01$).

absorption of an electromagnetic wave and transition to the regime at which the increment value is determined by the competition between two mechanisms of loss: the collision absorption of sound waves, which change slowly (by only $\sqrt{2}$ times) and the radiation escape from the interaction region (Fig. 5). With an increase in the loss, the position of the minimum is displaced toward larger scattering angles, which confirms the above arguments. This effect can be used for plasma diagnostics. The angular dependences obtained in this work differ both from known dependences in the approximation of homogeneous plasma and from the dependences obtained in a limited plasma, but without taking the propagation of sound waves into account.

In the experiments parametric instabilities can be observed in a wider range of parameters from microwaves to the optical range. We can estimate the possibility of the observation of this effect. An excited ion-sound wave should be weak decaying, which means the fulfillment of condition $2k_0V_S \ll \omega_{Li}$ [25] (ω_{Li}, the Langmuir frequency of ions). This relates the pumping-wave frequency and the density of electrons in plasma and can be expressed in several equivalent forms

$$\frac{\omega_{Pi}^2}{\omega_0^2} \gg \frac{4V_S^2}{c^2}, \quad \frac{\omega_{Pe}^2}{\omega_0^2} \gg \frac{4V_{Te}^2}{c^2},$$
$$n_e \gg \frac{4\omega_0^2}{c^2}\frac{kT_e}{4\pi e^2}, \quad \frac{4\omega_0^2}{c^2} r_{De}^2 \ll 1. \tag{9}$$

It follows from Eq. (9) that the density of electrons in the plasma is limited from below $n_e(\mathrm{cm}^{-3}) \gg 2.2 \times 10^6\, T_e(\mathrm{eV})\omega_0^2/c^2$. This condition also limits the pressure range of a neutral gas, as the electron density cannot exceed the density of neutral atoms (with a single ionization). Secondly, in a weakly ionized plasma the frequency of collisions of ions with neutral atoms should be low, which leads to the inequality $2k_0V_S \gg \nu_{in}$. This condition limits the range of the possible gas pressure from above

$$p_0(\mathrm{Torr}) \ll \frac{\omega_0}{\nu_{in}}\frac{V_s}{c} \approx \frac{\omega_0(s^{-1})}{3\times 10^4 \nu_{in0}}\sqrt{\frac{T_e(\mathrm{eV})}{A}}, \tag{10}$$

where A is the atomic mass of an ion and ν_{in0} is the collision frequency of an ion with a neutral atom at a pressure of 1 Torr. The third condition

$$\frac{\nu_{in}L}{\sqrt{cV_s}} \approx \sqrt{\frac{V_S}{c}\frac{L}{l_S}} < 1, \tag{11}$$

determines the dimension of the system at which the time of scattered waves energy removal by convection is less than the time of wave attenuation. It follows from Eq. (11) $L(\mathrm{cm}) \ll 16/(P(\mathrm{Torr})\sqrt{T_e(\mathrm{eV})/A})$. The condition of the applicability of geometric optics $L(\mathrm{cm}) \gg c/\omega_0$ also limits the range of possible pressures from below. At pulsed pumping fields the pulse duration τ should exceed the propagation time of an ion-acoustic wave by the scattering region $\tau \gg L/V_S$. The calculation shows that the above conditions in hydrogen plasma can be fulfilled in all discussed frequency ranges at certain plasma parameters and dimensions of the interaction region.

Finally, we will estimate an approximate amplitude of the electric field corresponding to the instability threshold in a hydrogen plasma for a wavelength of a pumping wave of 2 mm (in accordance with the above). Let dimension L equal 20 cm, the electron density be 10^{-12} cm^{-3}, the neutral gas pressure be 0.001 Torr, and the electron temperature be 5 eV. The frequency of an ion-acoustic wave during the backward scattering equals 140 MHz, and the threshold field in accordance with formula (7) equals 840 V/cm.

It should be noted that a similar effect can occur during decay instability with Langmuir and electromagnetic waves.

CONCLUSIONS

Brillouin scattering for a scattering region that is limited along the direction of the pumping-wave propagation and infinite in the transverse direction was considered. Calculations revealed that under these conditions a new effect can be observed: a decrease in the instability threshold under deviation of the scattering angle from π. This effect is related to the fact that the runaway time of the scattered radiation substantially increases in the geometry under consideration during scattering at an angle, which can compensate for a decrease in the coefficient of nonlinear feedback. The angular dependences obtained in the work differ both from known dependences in the approximation of homogeneous plasma and from the dependences obtained in the limited plasma, but without taking sound-wave propagation into account.

This effect is important in problems of plasma heating, as due to a lower threshold the stimulated scattering will occur at lower intensities of the pumping wave and the scattering will change the absorption coefficient and can be used for plasma diagnostics as well.

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